Opportunity Knocks: a Theoretical Perspective on B-Physics

Iain Stewart
MIT

SLUO, 2005
Weak Decays

- Test our understanding of QCD

\[ V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \]

CKM matrix

- Measure weak flavor physics of quarks

- Search for new physics

\[ b \rightarrow u e \bar{\nu}_e, \quad b \rightarrow s \bar{s}s \ldots \]
B Factory Breakthroughs

- \( \sin(2\beta) = 0.687 \pm 0.032 \) \( \mathcal{CP} \) in \( B \)-decays, consistency of SM

- Measurements of \( \alpha = 99^\circ \pm 12^\circ \) and \( \gamma = 63^\circ \pm 15^\circ \)
  now dominate other CKM constraints, bounds new physics in \( B-\bar{B} \) mixing

- Precision analyses: determination of \( |V_{cb}| = (41.5 \pm 0.7) \times 10^{-3} \),
  \( m_{1S}^b = (4.68 \pm 0.03) \) GeV, \( \overline{m}_c(m_c) = (1.224 \pm 0.057) \) GeV
  simultaneous fit to hadronic parameters, robust

- \( A_{CP}(K^-\pi^+) = -0.115 \pm 0.018 \) direct \( \mathcal{CP} \), existence of strong phases

- \( S_{\psi K} - S_{\eta'K_S} = 0.21 \pm 0.10 \) and \( S_{\psi K} - \langle S_{b \to s} \rangle = 0.18 \pm 0.06 \)
  not conclusive yet (these central values at \( 5\sigma \) would be new physics)

- More: \( |V_{ub}|, \Delta m_d, B \to X_s\ell^+\ell^-, B^0 \to D^0\pi^0, B \to \rho\gamma \ldots \)
constraints from angles dominate, will scale with statistics

other measurements test the SM, constrain new flavor physics

Think of $H_{weak} = \sum_{i=1}^{100} C_i O_i$ where SM relates the $C_i$ and all these connections need to be tested
Plan for this talk:

- $B \to X_s \gamma$, $B \to X_s \ell^+ \ell^-$
- $D \to K \ell \bar{\nu}$, $f_D, f_{D_s}$
- $f_B, f_{B_s}, B_d, B_b$
- $B \to \tau \bar{\nu}$
- $B \to \pi \ell \bar{\nu}$
- $B \to X_u \ell \bar{\nu}$
- $B \to D \pi$
- $B \to \eta' K^0$
- $B \to \rho \rho$ $B \to \pi \pi$
- $B \to K \pi$
- $B \to K^* \gamma$ $B \to \rho \gamma$

QCD tools

- Operator Product Expansion & Perturbative QCD
- Unquenched Lattice QCD
- Factorization Theorems for Weak Decays

- test SM
- test lattice QCD
- $\Delta m_d, \Delta m_s$
- $|V_{ub}|$
- test factorization
- test SM
- measure $\alpha, \gamma$, & test SM
- test SM
**Operator Product Expansion (I)**

- \( m_W, m_t \gg m_b \)

\[
H_{\text{weak}} = \frac{G_F}{\sqrt{2}} \sum_i \lambda_i C_i(\mu) O_i(\mu)
\]

\( \lambda^1 = \text{CKM}, \quad \lambda^1 = V_{ub}V_{ud}^* \)

Decays like \( B \to X_s\gamma \) & \( B \to K\pi \)

have contributions from \( \sim 12 \) operators

**perturbative QCD**
Operator Product Expansion (II)

- $m_b \gg \Lambda_{\text{QCD}}$

$$\Gamma = c^{(0)} f^{(0)} + \frac{1}{m_b} c^{(1)} f^{(1)} + \ldots$$

Heavy Quark Effective Theory

- $h_v, q$

Operator Product Expansion for Inclusive Decays

- Justifies free quark decay as leading approximation

$$\frac{\Lambda}{m_b} \simeq 0.1, \quad \alpha_s(m_b) \simeq 0.2$$

Subleading terms are crucial for precision phenomenology

B-meson

$\text{b}$
Unquenched Lattice QCD

\[ \det(\mathcal{D} + m) \neq 1 \]

nonperturbative QCD

Now:

- Focus on “Gold Plated Observables” for high precision
  - matrix elements with at most one hadron in initial and final state
  - at least 100 MeV below threshold, or small widths

- Simulate “real QCD”. Use \( nf=2+1 \) light flavors, quark masses \( m_q \) light enough for extrapolation with chiral perturbation theory (or PQChPT)

- Systematic/parametric estimates of uncertainties using effective field theory methods. eg. heavy quarks:
  - \( m_q \gg \Lambda_{QCD} \), NRQCD, Fermilab action, RHQ action

- Results for a broad spectrum of observables are obtained using common inputs
  \( \text{ChPT, PQChPT} \)

\( \uparrow \)
Factorization Theorems

Energetic Hadrons

eg. $E_\pi \gg \Lambda_{QCD}$

Soft-Collinear Effective Theory (SCET)

Introduce fields for infrared d.o.f.

collinear: $\xi_n, A^\mu_n$

soft: $h_v, q_s, A^\mu_s$

$\mathcal{L} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \ldots$

- Separate physics at different momentum scales
- Model independent, systematically improvable
Factorization Theorems

Energetic Hadrons

eg. \( E_\pi \gg \Lambda_{\text{QCD}} \)

\[
A = \int dz dx_i dk^+ T(z) J(z, x_i, k^+) \phi_1(x_1) \phi_2(x_2) \phi_B(k^+) + \ldots
\]

\( Q^2 \gg E \Lambda \gg \Lambda^2 \)
Inclusive Rare Decays
\[ B \rightarrow X_s \gamma \text{ } \& \text{ } B \rightarrow X_s \ell^+ \ell^- \]

- SM perturbative and nonperturbative effects are under control
- sensitive to new physics

3 steps

1) **Matching**

\[ H_W = \frac{G_F}{\sqrt{2}} V_{tb}^* V_{ts} \sum_{i=1}^{10} C_i(\mu) O_i(\mu) \] determine \( C_i(m_W) \)

2) **Running** (operator mixing)

\[ C_i(m_W) \rightarrow C_i(m_b) \]

\[ \mathcal{L} = \mathcal{L}_{QCD} + \mathcal{L}_{QED} \]

3) **Matrix elements** of \( O_i(\mu) \) with OPE at \( \mu \approx m_b \)
**Progress on NNLL calculations, a few entries still missing**

\[
B \to X_s \gamma
\]

\[
\begin{array}{cccc}
\text{Matching} & C_{1-6} & \text{tree} & 1L & 2L \\
 & C_{7,8} & 1L & 2L & 3L \\
\text{Running} & \hat{\gamma} & (1L \ 2L) & (2L \ 3L) & (3L \ 4L) \\
\langle O_{1-6} \rangle & 1L & 2L & 3L \\
\langle O_{7,8} \rangle & \text{tree} & 1L & 2L \\
\end{array}
\]

- **Matching**
  - \( C_{1-6} \)
  - \( C_{7,8} \)

- **Running**
  - \( \hat{\gamma} \) 

- **M. Elts.**
  - \( \langle O_{1-6} \rangle \)
  - \( \langle O_{7,8} \rangle \)

**Matching**

- Bobeth, Misiak, Urban
- Misiak, Steinhauser
- Haisch, Gorbahn, Gambinio
- Czakon et al.

**Running**

- Bieri, Greub, Steinhauser
- Greub, Hurth, Asatrian
- Blockland et al., Melnikov, Mitov
- Gambina, Gorbahn, Haisch
- Asatrian, Greub, Hurth
- Misiak, Steinhauser

\[
\frac{1}{(m_b)^k} \text{ corrections: } \text{Falk, Luke, Savage, Bauer}
\]

\[
\frac{1}{(m_c)^k} : \text{Voloshin, Khodjamirian, Ligeti, Randall, Wise, Grant, Morgan, Nussinov, Peccei, Buchalla, Isidor, Rey}
\]
Photon energy cut: $E_\gamma \geq E_0$

- $E_0 \geq 1.2 \text{ GeV}$ to avoid corrections where gluon or quark fragments into a photon
- $E_0 \leq 2.0 \text{ GeV}$ to keep it inclusive and avoid sensitivity to b-quark distribution function (region where standard OPE breaks down)

Usually argued that $E_0 = 1.6 \text{ GeV}$ suffices

Experiment:

- $E_0 \geq 2.0 \text{ GeV}$
- $E_0 \geq 1.8 \text{ GeV}$
- $E_0 \geq 1.9 \text{ GeV}$

Cut dependence can be systematized (uses SCET and OPE). Recently argued that $\alpha_s^2(m_b-2E_0)$ terms give an added $\sim 10\%$ uncertainty.

- Cleo'01
- Belle'04
- LP'05

On-resonance (this b-quark distn. is important for inclusive Vub determinations)
**Theory Summary**  \( (E_0 = 1.6\, \text{GeV}) \)

\[
\left. \text{Br}(B \to X_s \gamma) \right|_{E_\gamma > 1.6\, \text{GeV}} = 3.57 \times 10^{-4} \left[ 1 \pm 0.055 \frac{(m_c/m_b)}{\text{otherNNLO}} \pm 0.02 (C_{\ell\nu}) \pm 0.03 \alpha_s(m_Z) \pm 0.02 \text{Br}_{\text{semi}}^{\text{expt}} \pm 0.01 m_t \pm 0.01 \text{CKM} \right]
\]

\[
= (3.57 \pm 0.28) \times 10^{-4}
\]

\[
\left\{ = (3.47^{+0.46}_{-0.50}) \times 10^{-4} \right\} \quad \text{Neubert}
\]

These errors will be decreased by ongoing computations.

**Compare to Data**

\[
\text{HFAG JULY 2005}
\]

\[
B_{\text{Br}}^{\text{expt}} = (3.39^{+0.30}_{-0.27}) \times 10^{-4}
\]

**NLL**

Gambino, Misiak; Buras, Czarnecki, Misiak, Urban

Updated for LP’05 by M. Misiak

**strong constraints on BSM physics**

**improved data needed for comparison at NNLL**
\[ B \to X_s \ell^+ \ell^- \]

**LL:**
\[ C_9(\mu) = \frac{4}{9} \ln \frac{m_W^2}{m_b^2} + \mathcal{O}(\alpha_s) \]

**NLL:**
- Counting is like LL \( B \to X_s \gamma \)
  - Similar to LL, the NLO calculation corresponds to the NNLO calculation \( \bar{B} \to X_{s\ell^+\ell^-} \). 

**NNLL:**
- Counting is like NLL \( B \to X_s \gamma \)
  - Similar to NLL, the NLO calculation corresponds to the NNLO calculation \( \bar{B} \to X_{s\ell^+\ell^-} \). 

**Nonperturbative corrections:**
- Falk et al., Ali et al., Buchalla, Isidori, Rey
- Ghinculov, Hurth, Isidori, Yao

**NNLL:**
\[ Br_{\text{av}} (M_{\ell^+\ell^-} > 0.2 \text{ GeV}) = (4.46^{+0.98}_{-0.96}) \times 10^{-6} \]

**NNLL:**
\[ Br(B \to X_s \ell^+ \ell^-) = 4.17 \pm 0.70 \]

17% error
\[
\frac{d\Gamma^{B\to X_s\ell^+\ell^-}}{dq^2} = \left(\frac{\alpha_{\text{em}}}{4\pi}\right)^2 \frac{G_F^2 m_b^5 |V_{ts}^* V_{tb}|^2}{48\pi^3} (1 - \hat{s})^2 \left[\left(4 + \frac{8}{\hat{s}}\right) |\tilde{C}_7^{\text{eff}} (\hat{s})|^2 \right.
\]
\[
+ (1 + 2\hat{s}) \left( |\tilde{C}_9^{\text{eff}} (\hat{s})|^2 + |\tilde{C}_{10}^{\text{eff}} (\hat{s})|^2 \right) + 12 \text{Re} \left( \tilde{C}_7^{\text{eff}} (\hat{s}) \tilde{C}_9^{\text{eff}} (\hat{s})^* \right) + \frac{d\Gamma^{\text{Brem}}}{d\hat{s}} \right]
\]
\[
\hat{s} = \frac{q^2}{m_b^2}
\]

\( B \to X_s \ell^+ \ell^- \) constrains New Physics in a different way from \( B \to X_s \gamma \)
$B \to X_s \ell^+ \ell^-$ NNLL Spectrum

- Experiments remove backgrounds from $J/\Psi, \Psi'$
- Reduced theory uncertainty for:
  1. $1 \text{ GeV}^2 \leq q^2 \leq 6 \text{ GeV}^2$
  2. $14.4 \text{ GeV}^2 \leq q^2$

sensitive to different Wilson coefficients for new physics tests

---

$10^7 \left( \frac{d\text{Br}}{dq^2} \right) (\text{GeV}^{-2})$

eg. $\text{BR}_{\ell\ell} (1 \text{ GeV}^2 \leq q^2 \leq 6 \text{ GeV}^2) = [1.574 \pm 0.106 | M_t = 0.072 | m_b = 0.059 | M = 0.045 \pm 0.035 \text{BR}_{sl} = 0.001 | m_c] \times 10^{-6}$

---

NNLL spectrum in OPE

**model for long-distance $c\bar{c}$ contributions**

Kruger, Sehgal

10% total theory error
$B \rightarrow X_s \ell^+ \ell^-$  NNLL Forward - Backward Asymmetry

\[ \overline{A_{FB}}(q^2) = \left[ \frac{d\Gamma}{dq^2} \right]^{-1} \int_{-1}^{1} d\cos \theta \frac{d^2\Gamma}{dq^2 d\cos \theta} \text{sign}(\cos \theta) \]

Location of zero of the FB-Asymmetry tests the SM

\[ q_0^2 = (3.90 \pm 0.25) \text{ GeV}^2 \]  
\[ q_0^2 = (3.76 \pm 0.22_{\text{theory}} \pm 0.24_{\text{mb}}) \text{ GeV}^2 \]

So does other spectrum information

These spectra have not yet been exploited but will be
Lattice QCD
Sources of Uncertainty

- statistics from Monte Carlo
- $m_q$, chiral extrapolation
- $a$, action discretization
- $L$, finite volume
- $(\alpha_s)^k$, perturbative matching
- $\frac{1}{m_Q}, a$ corrections in matching

Unquenched Simulations

Wilson [nf=2: CP-PACS, JLQCD, QCDSF, UKQCD, qq+q, SPQcdR]
  [nf=2+1: CP-PACS / JLQCD]
  - expensive, chiral symmetry only recovered as $a \rightarrow 0$

Domain-wall [nf=2: RBC]
  - most expensive, exact chiral symmetry

Improved Staggered [nf=2+1: MILC]
  - fast, residual chiral symmetry,
  but 4 “tastes” for each flavor

\{ valence/sea $m_q$‘s down to 0.1 $m_s$
  ($m_\pi \approx 260\text{-}320 \text{ MeV}$) \}
“4-th root trick” for Staggered Fermions

\[ \det(\mathcal{D} + m) \rightarrow \det(\mathcal{D} + m)^{1/4} \] removes bad tastes, but Not Proven!

(HPQCD, UKQCD, MILC, Fermilab ‘03)

- tested at 3% level by comparison with mass spectra & light meson decay constants
- common input parameters
  \[ m_{\pi} \rightarrow m_u = m_d, \quad m_K \rightarrow m_s, \quad m_{D_s} \rightarrow m_c, \]
  \[ m_{\Upsilon} \rightarrow m_b, \quad m_{\Upsilon} - m_{\Upsilon'} \rightarrow \alpha_s(1/a) \]
- effect of unquenched calculation is clear

I’ll assume that the fourth rooted staggered fermion is valid
This will be tested by future data and other \((n_f=2+1)\) fermion formulations

CLEO\(_C\) Focus, BES test lattice QCD in D-decays then apply lattice QCD for physics at BaBar and Belle

\[ \begin{array}{c}
\text{CLEO}_C \\
\text{Focus, BES} \\
\text{test lattice QCD in D-decays} \\
\text{then apply lattice QCD for physics at BaBar and Belle}
\end{array} \]
\[ D \rightarrow K \ell \bar{\nu} , \quad D \rightarrow \pi \ell \bar{\nu} \]

\[
\langle K(p_K)|V^\mu|D(p_D)\rangle = f_+(q^2) \left( p_D^\mu + p_K^\mu - \frac{m_D^2 - m_K^2}{q^2} q^\mu \right) + f_0(q^2) \frac{m_D^2 - m_K^2}{q^2} q^\mu
\]

\[ D \rightarrow K\ell\nu \]

Shape agrees

- FNAL / MILC / HPQCD prediction prior to FOCUS result

\[ c \rightarrow s(d) \]
### Form Factor Normalization

<table>
<thead>
<tr>
<th></th>
<th>$f^D_{+\rightarrow K}(0)$</th>
<th>$\frac{f^D_{+\rightarrow \pi}(0)}{f^D_{+\rightarrow K}(0)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lattice</strong></td>
<td>0.73(3)(7)</td>
<td>0.87(3)(9)</td>
</tr>
<tr>
<td><strong>CLEO-C</strong></td>
<td></td>
<td>0.86(9)</td>
</tr>
<tr>
<td><strong>BES</strong></td>
<td>0.78(5)</td>
<td>0.93(20)</td>
</tr>
<tr>
<td><strong>FOCUS</strong></td>
<td></td>
<td>0.85(6)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Systematics</th>
<th>Fermilab/MILC/HPQCD errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>matching</td>
<td>&lt;1%</td>
</tr>
<tr>
<td>chiral extrapolation</td>
<td>2-3%</td>
</tr>
<tr>
<td>$q^2$ interp.</td>
<td>2%</td>
</tr>
<tr>
<td>finite a</td>
<td>9%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>10%</td>
</tr>
</tbody>
</table>

\[ \text{Br}(D^0 \rightarrow K \ell \nu) \times 10^2 \]
\[ \text{Br}(D^0 \rightarrow \pi^- \ell \nu) \times 10^3 \]

### Diagram

- Cleo*
- BES*
- Lattice*

* LP’05 update
* with PDG |Vcs|, |Vcd|
Normalization agrees
The $f_{D^+}$ Challenge!

$$D^+ \rightarrow \mu^+ \nu_{\mu} \quad \Gamma(D^+ \rightarrow \mu^+ \nu) = \frac{G_F^2 m_{D}}{8\pi} m_{\mu}^2 \left(1 - \frac{m_{\mu}^2}{m_{D}^2}\right)^2 f_{D^+}^2 |V_{cd}|^2$$

$$\langle 0|\bar{d} \gamma^\mu \gamma_5 c|D^+(p)\rangle = f_{D^+} p^\mu$$

<table>
<thead>
<tr>
<th></th>
<th>pre-LP'05</th>
<th>new at LP'05</th>
</tr>
</thead>
<tbody>
<tr>
<td>CP-PACS (prelim.)</td>
<td>20%</td>
<td>~ 13%</td>
</tr>
<tr>
<td>Fermilab/MILC/HPQCD (hep-lat/0506030)</td>
<td>24%</td>
<td>~ 8%</td>
</tr>
<tr>
<td>CLEO-C</td>
<td>22%</td>
<td>~ 8%</td>
</tr>
</tbody>
</table>

Errors decreased by factor of 3
$f_{D+}$ \quad $n_f = 2 + 1$

Fermilab/MILC/HPQCD

- A test for light quarks & the staggered formalism.

Use staggered ChPT analog of

$$\Delta f_D^{\text{chiral log}} = -\frac{3}{4} (1 + 3g^2) \frac{m^2_{\pi}}{(4\pi f)^2} \ln \frac{m^2_{\pi}}{\mu^2}$$

- Shift is caused by including the $O(a^2)$ terms in non-log part of the chiral extrapolation (main reason for decrease from prelim. to final)

- Largest uncertainty is from light quark discretization & ChPT (but its only 6% !)
Results

\[ n_f = 2 \]
\[ n_f = 2 + 1 \]

\[ f_{D^+} = 202 \pm 12^{+20}_{-25} \text{ MeV} \]
\[ f_{D^+} = 201 \pm 3 \pm 17 \text{ MeV} \]
\[ f_{D^+} = 223 \pm 16^{+7}_{-9} \text{ MeV} \]

Good Agreement
new LP’05 HPQCD results (preliminary, nf=2+1):

\[
\frac{f_{B_s}}{f_B} = 1.20 \pm 0.02 \pm 0.01
\]

\[
f_B = (218 \pm 9 \pm 21) \text{ MeV}
\]

\[
f_{B_s} = (260 \pm 7 \pm 28) \text{ MeV}
\]

consistent with 2003: \( f_{B_s} = (260 \pm 7 \pm 28) \text{ MeV} \)
\( \Delta m_s \) \& \( \Delta m_d \) Constraints with Unquenched LQCD

\[
\Delta m_d = C_{\text{short}} m_{B_d} f_B^2 \hat{B}_d |V_{td} V_{tb}^*|^2
\]

\[
\frac{\Delta m_d}{\Delta m_s} = \frac{m_{B_d}}{m_{B_s}} \frac{f_B^2}{f_{B_s}^2} \frac{\hat{B}_d}{\hat{B}_s} |V_{td}|^2 \propto \xi^2
\]

\[
\alpha \beta \gamma \rho \eta
\]

excluded area has CL > 0.95

CKM fit data: Improvement is from increased central value and decreased statistical error

\[
\Delta m_d \propto \frac{1}{f_B^2}
\]

\[
\xi = 1.21 \pm 0.022^{+0.035}_{-0.014}
\]

\[
f_B \sqrt{\hat{B}_d} = (246 \pm 11 \pm 25) \text{ MeV}
\]

\[
f_{B_s} \sqrt{\hat{B}_s} = (296 \pm 9 \pm 33) \text{ MeV}
\]
**Constraints with Unquenched LQCD**

\[ \Delta m_d = C_{\text{short}} m_{B_d} f_B^2 \hat{B}_d |V_{td} V_{tb}^*|^2 \]

\[ \frac{\Delta m_d}{\Delta m_s} = \frac{m_{B_d}}{m_{B_s}} \left( \frac{f_B^2}{f_{B_s}^2} \right) \frac{\hat{B}_d}{\hat{B}_s} \frac{|V_{td}|^2}{|V_{ts}|^2} \]

\[ \propto [(1 - \bar{\rho})^2 + \bar{\eta}^2] \]

Assume \( \Delta m_s \) was measured.

\[ \hat{B}_d = 1.271(41)(+85\text{-}94) \]

\[ n_f = 2 \]

\[ \hat{B}_s = 1.017(16)(+56\text{-}17) \]

\[ \xi = 1.21 \pm 0.022^{+0.035}_{-0.014} \]

\[ f_B \sqrt{\hat{B}_d} = (246 \pm 11 \pm 25) \text{ MeV} \]

\[ f_{B_s} \sqrt{\hat{B}_s} = (296 \pm 9 \pm 33) \text{ MeV} \]

Assume \( \Delta m_s \) was measured.

New lattice errors on \( \xi \) reduced the width of the green band by \( \sim 50\% \).
\[ \Gamma(B \to \tau\nu) = \frac{G_F^2 m_B}{8\pi} m_\tau^2 \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \]

no direct measurement yet, but getting close

\[ Br(B^+ \to \tau^+\nu_\tau) < 2.6 \times 10^{-4} (90\%) \quad \text{Babar (LP'05)} \]
\[ Br(B^+ \to \tau^+\nu_\tau) < 1.8 \times 10^{-4} (90\%) \quad \text{Belle (LP'05)} \]

Use it:
- with \( f_B \), a clean result for \( |V_{ub}| \)
- with \( V_{ub} \), it reduces uncertainty associated with \( f_B \)
  eg. \( \Gamma(B \to \tau\nu)/\Delta m_d \) is independent of \( f_B \)
$|V_{ub}|$
$V_{ub}$

$B \to \pi \ell \bar{\nu}$

$\frac{d\Gamma}{dq^2}(\bar{B}^0 \to \pi^+ \ell \bar{\nu}) = \frac{G_F^2 |\vec{p}_\pi|^3}{24\pi^3} |V_{ub}|^2 |f_+(q^2)|^2$

**Precision**

<table>
<thead>
<tr>
<th>$q^2$ (GeV$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>1.0</td>
</tr>
</tbody>
</table>

$\vec{p}_\pi \ll 1/a$

**Rate is smaller at large $q^2$**

**Uncertainty from theory dominates.**

Average from Cleo, Belle, Babar:

<table>
<thead>
<tr>
<th>B($B^0 \to \pi^+ \ell \bar{\nu}$) [$\times 10^{-4}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>BABAR SL tag: $B^+ \to \pi^0 l^+ \nu$</td>
</tr>
<tr>
<td>BABAR Breco tag: $B^+ \to \pi^0 l^+ \nu$</td>
</tr>
<tr>
<td>BABAR Breco tag: $B^0 \to \pi^0 l^+ \nu$</td>
</tr>
<tr>
<td>Belle SL tag: $B^0 \to \pi^0 l^+ \nu$</td>
</tr>
<tr>
<td>CLEO untagged: $B \to \pi^0 l^+ \nu$</td>
</tr>
<tr>
<td>BABAR untagged: $B \to \pi^0 l^+ \nu$</td>
</tr>
<tr>
<td>Average: $B^0 \to \pi^0 l^+ \nu$</td>
</tr>
</tbody>
</table>

$\chi^2$/dof = 11.2/6 (CL = 8.3%)
Method 1: Model Independent

Pure Lattice QCD

\[ q^2 \geq 16 \text{ GeV}^2 \]

![Graph showing model independent pure lattice QCD results for \( B \to \pi \) with different ensembles and statistics.](image)

- Systematics: 4-6%
- Perturbative matching: 9%
- Chiral extrapolation: 4%
- Action discretization: 2%
- Matching \( a, 1/m_Q \): 5%
- Total: 11%

Statistics

- Fermilab/MILC errors:
  - Matching: 1%
  - Chiral extrapolation: 4%
  - \( q^2 \) interp.: 4%
  - Finite \( a \): 9%
  - Total: 11%
Comparison of lattice calculations

**Preliminary:** HPQCD (hep-lat/0408019) and Fermilab/MILC (hep-lat/0409116)

**Systematics**

<table>
<thead>
<tr>
<th>Systematics</th>
<th>HPQCD errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>perturbative matching</td>
<td>9%</td>
</tr>
<tr>
<td>chiral extrapolation</td>
<td>4%</td>
</tr>
<tr>
<td>action discretization</td>
<td>2%</td>
</tr>
<tr>
<td>matching $a, 1/m_Q$</td>
<td>5%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>11%</strong></td>
</tr>
</tbody>
</table>

**Systematics**

<table>
<thead>
<tr>
<th>Systematics</th>
<th>Fermilab/MILC errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>matching</td>
<td>1%</td>
</tr>
<tr>
<td>chiral extrapolation</td>
<td>4%</td>
</tr>
<tr>
<td>$q^2$ interp.</td>
<td>4%</td>
</tr>
<tr>
<td>finite a</td>
<td>9%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>11%</strong></td>
</tr>
</tbody>
</table>

$q^2 \geq 16 \text{ GeV}^2$

**HFLAG LP’05**

\[
10^3 \times |V_{ub}| = 3.75 \pm 0.27^{+0.64}_{-0.42}
\]

**FNAL**

\[
10^3 \times |V_{ub}| = 4.45 \pm 0.32^{+0.69}_{-0.47}
\]

**HPQCD**

\[
10^3 \times |V_{ub}| = 4.1 \pm 0.32^{+0.69}_{-0.42}
\]

**My Average for this method:**

\[
10^3 \times |V_{ub}| = 4.1 \pm 0.32^{+0.69}_{-0.42}
\]

**Statistics**

- 4-6%
- ∼ 8%

Total error **16%**

Statistics **16%**

Statistics **4-6%**

Statistics **4-6%**

Statistics **4-6%**
Method II:

**Light-cone QCD sum-rules**

compute form factors for small $q^2$

Error Analysis for $f_+(0)$  
Ball, Zwicky

$$f_+(0) = 0.258 \pm 0.031$$

Babar (LP’05)  $q^2 < 16 \text{ GeV}^2$

\[
\begin{align*}
\text{exp.} & \quad \text{theory} \\
10^3 \times |V_{ub}| &= 3.27 \pm 0.25^{+0.54}_{-0.37} \\
16\% \text{ total error}
\end{align*}
\]
Method III:

**Lattice & QCD Dispersion Relations**

i) Lattice qcd results at large $q^2$

ii) chiral perturbation theory at $q^2_{\text{max}}$

iii) expt. spectra for information at low $q^2$

(Babar updated at LP'05)

& SCET constraint from $B \to \pi\pi$ at $q^2 = 0$

iv) QCD dispersion relations to constrain the form factors shape

---

Model Independent

Bourrely et al.,
Boyd, Grinstein, Lebed, Savage;
Lellouch; Fukunaga, Onogi;
Arnesen, Grinstein, Rothstein, I.S.
\[ z(t, t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}} \]

\[ t_\pm = (m_B \pm m_\pi)^2 \]

\[ P(t)\phi(t)f(t) = \sum_{n=0}^{\infty} a_n z^n \]

Blaschke Factor: remove pole at \( t = m_B^2 \)

Outer function: phase space, Jacobian, \( \chi^{(0)} \) in QCD

\[ \sum_{n} a_n^2 \leq 1 \]

Pick \( t_0 = 0.65 \, t_- \) then

\[-0.34 \leq z \leq 0.22\]

Strategy: use input points to fix first few a’s
vary higher a’s to determine uncertainty
**** Plots for Fit Results

\[ (1 - \hat{q}^2)f_+(q^2) \]

- **ChPT pt.**
- **SCET pt.**
- **Lattice points**

---

\[ m_B^2 \frac{d\Gamma}{dq^2} \]

- **with SCET point**
- **without SCET point**

---

(1 - \hat{q}^2)f(q^2)

- **f=f_+**
- **f=f_0**

Dispersion relations show there is a lot of freedom for a pure extrapolation of lattice data

---

- **expt. spectrum prefers a larger form factor in \( \sim 5-10 \text{ GeV}^2 \) region**
- **Here the SCET point constrains the spectrum, but does not change the determination of Vub**
$\chi^2$ fits to data & input pts. with dispersion relations

$\chi^2/(dof) \sim 1.0$  

$10^3 \times |V_{ub}| = 3.72 \pm 0.52$  

FNAL

$10^3 \times |V_{ub}| = 4.11 \pm 0.52$  

HPQCD

My Average for this method:

$10^3 \times |V_{ub}| = 3.92 \pm 0.52$  

13% total error  

(4% expt.)

---

Type of Error | Variation From | $\delta|V_{ub}|q^2$
---|---|---
Input Points | 1-$\sigma$ correlated errors | ±13%
Bounds | $F_+ \text{ versus } F_-$ | < 1%
m_pole | 4.88 ± 0.40 | < 1%
OPE order | 2 loop → 1 loop | < 1%

fit also gives:  

$f_+(0) = 0.25 \pm 0.06$

similar to sum-rules

This includes the information in the pure lattice method
Inclusive Vub

\[ |V_{ub}|^{\text{incl}} = (4.38 \pm 0.33) \times 10^{-3} \]

Lots of theory work, eg.
- Event generator Neubert, Lange, Paz
- Subleading shape functions

\[ |V_{ub}|^{\text{incl}} = (3.53^{+0.22}_{-0.21}) \times 10^{-3} \]

(CKMfitter LP'05)

Tension with \( \sin(2\beta) \)?

\[ |V_{ub}|^{\text{incl}} = (3.53^{+0.22}_{-0.21}) \times 10^{-3} \]

(CKMfitter LP'05)

\[ |V_{ub}|^{\text{incl}} = (4.38 \pm 0.33) \times 10^{-3} \]

\[ |V_{ub}|^{\text{excl}} = (3.92 \pm 0.52) \times 10^{-3} \]
Nonleptonic Decays
Motivation Going Forward

- So far we’ve been talking about precision theory $\lesssim 10\%$
- Now we will turn to cases where the expansion is worse, $\sim 20\%(?)$
- But the odds are higher! We can look for new physics in many channels, where the sensitivity appears in different ways.
- Need to know what the SM expectation is for $\text{Br}$ and $\text{CP-Asymmetries}$
- Some decays are “clean” (small uncertainty) factorization can provide error estimates
\[ \bar{B}^0 \rightarrow D^0 M^0 \]

**Testing Factorization and SCET**

Mantry, Pirjol, I.S.

Blechman, Mantry, I.S.

\[ A_{00}^{D(*)\pi} = N_0^{(*)} \int dx \, dz \, dk_1^+ \, dk_2^+ \, T^{(i)}(z) \, J^{(i)}(z, x, k_1^+, k_2^+) \, S^{(i)}(k_1^+, k_2^+) \, \phi_\pi(x) \]

\[ + A_{\text{long}}^{D(*)\pi} \]

\[ \frac{\Lambda}{E_M} \quad \& \quad \frac{1}{N_c} \quad \text{suppressed} \]

**Predict**

equal strong phases \( \delta(DM) = \delta(D^*M) \)

equal amplitudes \( A(D^*M) = A(DM) \)

**Find**

\[ \delta(D\pi) = 30.4 \pm 4.8^\circ \]

\[ \delta(D^*\pi) = 31.0 \pm 5.0^\circ \]

\[ \left| \frac{A(D^*M)}{A(DM)} \right| \]

**Without factorization**

predictions spoiled by \( \mathcal{O}\left(\frac{E_M}{m_c}\right) = \mathcal{O}(1) \) effects

\[ \delta(D\pi) = 30.4 \pm 4.8^\circ \]

\[ \delta(D^*\pi) = 31.0 \pm 5.0^\circ \]
$B \rightarrow M_1 M_2$

**Methods**

- SU(2), isospin symmetry \( \frac{m_{u,d}}{\Lambda} \approx 0.02 \)
- SU(3) symmetry \( \frac{m_s}{\Lambda} \approx 0.3 \)
- Factorization \( \Lambda^2 \ll E \Lambda \ll E^2, m_b^2 \)

**corrections \( \sim 20\% \)**

not great precision, but sufficient for large new physics signals (and improvable)

- SU(2), isospin symmetry
- SU(3) symmetry
- Factorization

**sizeable charm loops?**

Ciuchini et al, Colangelo et al

**Large Annihilation**

\( C_1 \frac{\Lambda}{E} \) competes

- many authors
  - classic: Gronau, London
  - many authors
    - Rosner, Lipkin, ...
  - Chay, Kim
  - Bauer, Pirjol, Rothstein, I.S.

**k_{\perp}**

Factorization

Keum, Li, Sanda, Lu et al.

(appears to be a good model for soft physics)
Factorization (with SCET)

**Factorization at** $m_b$  
Bauer, Pirjol, Rothstein, I.S.

**Nonleptonic**  
$B \to M_1 M_2$

$A(B \to M_1 M_2) = A^{c\bar{c}} + N \left\{ f_{M_2} \zeta^{BM_1}_B \int du T_{2\xi}(u) \phi^{M_2}(u) + f_{M_2} \int dudz T_{2J}(u, z) \zeta^{BM_1}_J(z) \phi^{M_2}(u) + (1 \leftrightarrow 2) \right\}$

**Form Factors**  
$B \to$ pseudoscalar: $f_+, f_0, f_T$  
$B \to$ vector: $V, A_0, A_1, A_2, T_1, T_2, T_3$

\[
f(E) = \int dz \, T(z, E) \zeta^{BM}_J(z, E) + C(E) \zeta^{BM}(E)\]

"hard spectator", "factorizable"

"soft form factor", "non-factorizable"

**Factorization at** $\sqrt{E\Lambda}$

expansion in $\alpha_s(\sqrt{E\Lambda})$

Beneke, Feldmann  
Bauer, Pirjol, I.S.  
Becher, Hill, Lange, Neubert

$\zeta^{BM}_J(z) = f_M f_B \int dx \int_0^1 dk^+ J(z, x, k^+, E) \phi_M(x) \phi_B(k^+)$

$\zeta^{BM} = ?$ (left as a form factor)
Choose some reasonable values for hadronic parameters. Test Qualitative Agreement with Factorization

QCDF: Buchalla et al.; Neubert, Beneke

pQCD: Keum, Li, Sanda \( (k_\perp) \)

(NOTE: some power suppressed terms included as well)
\[ \mathcal{B}(B \rightarrow K\pi, \pi\pi, KK) \]

**Legend:**
- CLEO
- Belle
- BABAR
- CDF
- PDG2004
- New Avg.

**Data Points:**
- \( K^0\bar{K}^0 \)
- \( K^+K^- \)
- \( K^+\bar{K}^0 \)
- \( \pi^0\pi^0 \)
- \( \pi^+\pi^0 \)
- \( \pi^+\pi^+ \)
- \( K^0\pi^0 \)
- \( K^+\pi^0 \)
- \( K^-\pi^- \)
- \( K^0\pi^+ \)

**Theory Models:**
- pQCD
- QCDF (default)
- QCDF (S4)

**Notes:**
- New data
- Pattern is reproduced
- Note: I did not add theory error estimates here
A Few Channels

Redundant measurements in different channels allow us to probe for new physics.
Isospin Analysis

$B \rightarrow \rho\rho$

( $\rho \parallel \rho \parallel$ dominates as factorization predicts A. Kagan)

Parameters = 6

Observables = 6

$\gamma + 5$ hadronic

$B \rightarrow \rho^0\rho^0$ channel is not measured, but strong experimental bound forbids sizeable penguins

$\alpha_{\rho\rho} = 96^\circ \pm 13^\circ$

$I = 1$ contributions

Falk et al.
$B \to \pi\pi$  \hspace{1cm} \textbf{Isospin Analysis}

Known strong isospin breaking effects are small
\[ \delta \alpha \sim 2^\circ \quad \text{Gardner; Gronau, Zupan} \]

Problem is precision of direct CP - Asymmetry for neutral pions

\[ C_{\pi^0\pi^0} = -0.28 \pm 0.39 \]

(Belle & Babar)

\[ C_{\pi^0\pi^0} = -0.28 \pm 0.39 \quad \text{(Belle & Babar)} \]

Worth remembering: more theory input/less fit parameters means more ways to test for new physics

eg. can’t see new physics in $I = 0$ amplitudes with the isospin analysis

Baek, Botella, London, Silva
Definitions:

\[
A(\bar{B}^0 \to \pi^+ \pi^-) = e^{-i\gamma} |\lambda_u| T - |\lambda_c| P \\
A(\bar{B}^0 \to \pi^0 \pi^0) = e^{-i\gamma} |\lambda_u| C + |\lambda_c| P \\
\sqrt{2}A(B^- \to \pi^0 \pi^-) = e^{-i\gamma} |\lambda_u| (T + C)
\]

|\lambda_{c,u}| = CKM factors , take \(\beta\) known

Data \[\rightarrow\] Significant \(P\), “penguins”,

Large \(C\), “color suppressed amplitude”
\[ Br(B \rightarrow \pi^0\pi^0) = 1.45 \pm 0.29 \quad \text{is large} \quad \text{(a hot topic in 2003)} \]

\[ \text{NOT a contradiction with factorization.} \]

**Why?**

- **if** \( \zeta_J^{B\pi} \sim \zeta^{B\pi} \), then a term \( \frac{C_1}{N_c} \langle \bar{u}^{-1} \rangle_{\pi} \zeta_J^{B\pi} \) in the factorization theorem ruins color suppression and explains the rate

\[ \zeta^{B\pi} \gg \zeta_J^{B\pi} \quad \text{this Br is sensitive to power corrections} \]

- **In the future:** determine parameters using improved data on the \( B \rightarrow \pi \ell \bar{\nu} \) form factor at low \( q^2 \) to provide a check.
• Power counting says Penguins can’t be TOO big and their strong phase should not be TOO large (assume factorization gets the sign right)

\[ \left| \frac{P}{T} \right| \leq 1 \]

\[ -\frac{\pi}{2} \leq \arg \left( \frac{P}{T} \right) \leq \frac{\pi}{2} \]

Removes discrete ambiguities
Factorization predicts a Flat Tree Triangle

$\epsilon \sim 0, \tau(t) \sim 0$

Use this to get $\alpha$ without $C^{\pi^0\pi^0}$.

$$\epsilon = \text{Im} \left( \frac{C}{T} \right) = O \left( \alpha_s(m_b), \frac{\Lambda}{E} \right) \lesssim 0.2$$

From: Grossman, Hoecker, Ligeti, Pirjol

$$\gamma = \pi - \beta^{\text{expt}} - \alpha$$

for $\alpha \sim 90^\circ$ $\epsilon = 0.2 \leftrightarrow \tau(t) \sim 5^\circ$

$\epsilon = 0.4 \leftrightarrow \tau(t) \sim 10^\circ$
\[ V_{ub} \]

\[ \gamma \]

Global fit

Inclusive

from $\beta$

$\gamma^{\pi\pi}$ SCET

$\gamma^{DK}$, $\alpha_{\rho\rho}$

\[ \frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \]

\[ (\rho, \eta) \]

\[ (0, 0) \]

\[ (1, 0) \]
Is there a $K$-$\pi$ CP Puzzle?

- Br sum rule:

$$R(\pi^0 K^-) - \frac{1}{2} R(\pi^- K^+) + R(\pi^0 K^0) = O(\epsilon^2)$$

$$0.094 \pm 0.073 = O(\epsilon^2) < 0.03$$

no puzzle here yet

- Direct-CP sum rule:

$$\Delta(\bar{K}^0\pi^0) - \frac{1}{2} \Delta(K^+\pi^-) + \Delta(K^+\pi^0) = O(\epsilon^2)$$

$$0.058 \pm 0.070 = O(\epsilon^2) < 0.007$$

no puzzle here yet

Expand in

$$\epsilon = \left| \frac{V_{us} V_{ub}}{V_{cs} V_{cb}} \right| \frac{T}{P} , \left| \frac{V_{us} V_{ub}}{V_{cs} V_{cb}} \right| \frac{C}{P} , \frac{P_{ew}(t,c)}{P}$$

Lipkin, many authors

$$R(f) = \frac{\Gamma(B \to f)}{\Gamma(\bar{B}^0 \to \pi^- \bar{K}^0)}$$

Gronau, Rosner

$$\Delta(f) = \frac{A_{CP}(f) \Gamma_{CP}^{\text{avg}}(f)}{\Gamma_{CP}^{\text{avg}}(\pi^- \bar{K}^0)}$$

my estimate from factorization in SCET
SU(3), global fits to data

12 parameters, 18 predictions

\( \pi \pi, KK, \pi \eta, \pi \eta' K \pi, K \eta, K \eta' \)

\[ \gamma = 61^\circ \pm 11^\circ \text{ agrees with global fit} \]

\( Br(K^+\pi^-), Br(K^0\pi^0), A_{CP}(K^0\pi^0) \)

hints of a puzzle?

SCET based fit

Bauer, Rothstein, I.S. (to appear)

\( \gamma = 59^\circ \text{ fixed} \)

6 parameters + 2 fixed by SU(3)

\( Br(K^+\pi^-), A_{CP}(K^-\pi^0) \)

hints of a puzzle?

Chiang, Gronau, Luo, Rosner, Suprun

better agreement when one adds new Babar data

give \( \Delta \chi^2 = (2.7, 5.9, 2.9) \)

see also Buras et al.; Kim et al.

\( \chi^2 \)

Global PP fit

pre-LP'05 data

Branching ratio \(\times 10^5\)

\( K^0 \pi^0 \)

\( K^- \pi^0 \)

\( K^+ \pi^- \)

\( K^0 \pi^- \)

\( \pi^0 \pi^0 \)

\( \pi^0 \pi^0 \)

\( \pi^+ \pi^- \)

\( \pi^+ \pi^- \)

\( A(\pi^0 \pi^0) \)

\( A(K^- \pi^0) \)

\( A(K^0 \pi^0) \)

\( A(K^0 \pi^-) \)

\( A(K^+ \pi^-) \)

\( C(\pi^+ \pi^-) \)

\( S(\pi^+ \pi^-) \)

\( \Delta \) Theory

\( \ast \) Data

\( \gamma \)
Some Nonleptonic Decays are Clean

**B → ψK, ...**

\[ A = V_{cb} V_{cs}^* (T + P_c - P_t) + V_{ub} V_{us}^* (P_u - P_t) \]

\[ \lambda^2 \]

\[ \lambda^4 \]

 Corrections: \(|\bar{A}/A| \neq 1, \epsilon_K \neq 0, \Delta \Gamma_B \neq 0\) \hspace{1cm} \text{few} \times 10^{-3}

\[ \sin(2\beta) \) with accuracy \lesssim 1\%

**B → η'K, ...**

\[ A = V_{cb} V_{cs}^* (P_c - P_t) + V_{ub} V_{us}^* (P_u - P_t) \]

\[ \lambda^2 \]

\[ \lambda^4 \]

 In SM expect: \(S_{η'K} - S_{ψK}, C_{η'K} \lesssim 0.05\)

 Dominated by loops or rescattering, more sensitive to new physics

**b → c\bar{c}s**

**b → q\bar{q}s**
$b \to c\bar{c}s$ vs. $b \to q\bar{q}s$

\begin{align*}
\sin(2\beta_{\text{eff}}) / \sin(2\phi_{1 \text{eff}}) & \quad \text{HFAG LP 2005 (PRELIMINARY)} \\
\text{World Average} & = 0.69 \pm 0.03 \\
\text{BaBar} & = 0.50 \pm 0.25 \pm 0.04 \\
\text{Belle} & = 0.44 \pm 0.27 \pm 0.05 \\
\text{Average} & = 0.47 \pm 0.19 \\
\text{BaBar} & = 0.30 \pm 0.14 \pm 0.02 \\
\text{Belle} & = 0.62 \pm 0.12 \pm 0.04 \\
\text{Average} & = 0.48 \pm 0.09 \\
\text{BaBar} & = 0.95 \pm 0.32 \pm 0.10 \\
\text{Belle} & = 0.47 \pm 0.36 \pm 0.08 \\
\text{Average} & = 0.75 \pm 0.24 \\
\text{BaBar} & = 0.35 \pm 0.33 \pm 0.04 \\
\text{Belle} & = 0.22 \pm 0.47 \pm 0.08 \\
\text{Average} & = 0.31 \pm 0.26 \\
\text{BaBar} & = 0.50 \pm 0.34 \pm 0.12 \\
\text{Belle} & = 0.95 \pm 0.53 \pm 0.15 \\
\text{Average} & = 0.63 \pm 0.30 \\
\text{BaBar} & = 0.41 \pm 0.18 \pm 0.07 \pm 0.11 \\
\text{Belle} & = 0.60 \pm 0.18 \pm 0.04 \pm 0.12 \\
\text{Average} & = 0.51 \pm 0.14 \pm 0.08 \\
\text{BaBar} & = 0.63 \pm 0.28 \pm 0.04 \\
\text{Belle} & = 0.58 \pm 0.36 \pm 0.08 \\
\text{Average} & = 0.61 \pm 0.23 \\
\end{align*}

largest deviations: $\eta' K_S (2\sigma)$ clean average (without theory error):

$S_{\psi K} - \langle S_{b \to s} \rangle = 0.18 \pm 0.06$
### Theory Uncertainty

<table>
<thead>
<tr>
<th>Dominant process</th>
<th>(f_{CP})</th>
<th>SM predictions for ((-\eta f_{CP} S f_{CP} - \sin 2\beta))</th>
<th>B.H.N.R.</th>
<th>Beneke</th>
<th>(\sin 2\beta_{eff})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b \to c\bar{c}s)</td>
<td>(\psi K_S)</td>
<td>&lt; 0.01</td>
<td></td>
<td></td>
<td>+0.687 ± 0.032</td>
</tr>
<tr>
<td>(b \to s\bar{q}q)</td>
<td>(\eta' K^0)</td>
<td>&lt; 0.05</td>
<td>+0.01^{+0.01}_{-0.02}</td>
<td>+0.01^{+0.01}_{-0.01}</td>
<td>+0.48 ± 0.09</td>
</tr>
<tr>
<td></td>
<td>(\phi K^0)</td>
<td>&lt; 0.05</td>
<td>+0.02^{+0.01}_{-0.01}</td>
<td>+0.02^{+0.01}_{-0.01}</td>
<td>+0.47 ± 0.19</td>
</tr>
<tr>
<td>(K^+ K^- K_S)</td>
<td>(\sim 0.15)</td>
<td></td>
<td></td>
<td></td>
<td>+0.51 ± 0.17</td>
</tr>
<tr>
<td>(K_S K_S K_S)</td>
<td>(\sim 0.15)</td>
<td></td>
<td></td>
<td></td>
<td>+0.61 ± 0.23</td>
</tr>
<tr>
<td>(\pi^0 K_S)</td>
<td>(\sim 0.15)</td>
<td>+0.06^{+0.04}_{-0.03}</td>
<td>+0.07^{+0.05}_{-0.04}</td>
<td></td>
<td>+0.31 ± 0.26</td>
</tr>
<tr>
<td>(f^0 K_S)</td>
<td>(\sim 0.25)</td>
<td>+0.19^{+0.06}_{-0.14}</td>
<td>+0.13^{+0.08}_{-0.08}</td>
<td></td>
<td>+0.63 ± 0.30</td>
</tr>
<tr>
<td>(\omega K_S)</td>
<td>(\sim 0.25)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

estimates* by Ligeti (see also Grossman et al., SU(3) bounds)

Buchalla, Hiller, Nir, Raz (BHNFR) and Beneke use factorization (QCDF inputs)
$B \rightarrow K^*\gamma$ & $B \rightarrow \rho\gamma$

Theory based on Factorization Formula:

$$\langle V_\gamma|Q_i|B \rangle = T_i^I F_V + \int dx dk \ T_i^{II}(x,k) \ \phi_B(k)\phi_V(x) + O(\frac{C_1}{m_b})$$

$$\xi = \frac{F_{K^*}}{F_{\rho}}$$

$$\xi = 1.25 \pm 0.20 \quad \text{(Ball, Zwicky; sum-rules)}$$

$$\xi = 1.1 \pm 0.1 \quad \text{(Becirevic, Mescia; lattice+extrapolation to small } q^2 )$$

$$\text{SU(3) violation}$$

$$\xi = \frac{\text{Br}(B \rightarrow \rho^0\gamma)}{\text{Br}(B \rightarrow K^*\gamma)} = \frac{1.023}{2} \left| \frac{V_{td}}{V_{ts}} \right|^2 \xi^{-2} \left[ 1 + 2(\text{ckm})\delta a \right] \left| \frac{a_i^c(\rho)}{a_7^c(K^*)} \right|^2$$

$$\text{small}$$

Factorization & Phenomenology


Ali, Lunghi, Parkhomenko

Bosch, Buchalla

SCET ($K^*\gamma$)

Chay, Kim;

Becher, Hill, Neubert

from Bosch,Buchalla

$$\text{CKM}'05$$

$$1.2 \pm 0.1$$

conservative ?
Neutral flavor changing process

\[ \text{Br}(B \to \rho^0 \gamma) = (1.17^{+0.35+0.09}_{-0.31-0.08}) \times 10^{-6} \]  
(Belle LP’05)

\[ \sigma(\xi) = 0.2 \]  
curve doubles theory error estimate

Currently agrees with global fit

\[ b \to d \gamma \]  
WA used (Babar, Belle)
Theory Summary

- **Radiative Decays**
  - progress on understanding and reducing the QCD uncertainties, \( B \to X_s \ell^+ \ell^- \) constraints on new physics

- **Lattice QCD**
  - new \( f_D \) agrees with new Cleo-C result
  - new staggered \( f_B, f_{BS}/f_B \) improves the \( \Delta m_d \) constraint
  - small theory uncertainty for \( \Delta m_s \) constraint

- **Lattice QCD & Continuum methods**
  - towards a precision determination of \( V_{ub} \)

- **Factorization Theorems**
  - New tools developed, progress in understanding Nonleptonic B-Decays, new strategies for \( \alpha \)
Looking into the Future
at B-factories

- improved determination of $\alpha$, $\beta$, $\gamma$
- clarify agreement / disagreement between $S_{\eta'K_S}$, $S_{\phi K_s}$, and $\sin(2\beta)$
- precision determination of $|V_{ub}|$
- match theoretical limits for sensitivity in $B \to X_s\gamma$ and $B \to X_s\ell^+\ell^-$
- observation of $B \to \rho\gamma$ and $B \to \tau\nu$
- Sort out puzzles in $B \to \pi\pi$ and $B \to K\pi$
- Approach SM predictions in semileptonic $A_{CP}$ and $S_{K^*\gamma}$
- .... and of course, the unexpected.