Hall Element Angle Calibrations For The Delta Undulator

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Abstract

One technique used to characterize the Delta undulator is to measure in the bore using an array of Hall probes. Each Hall element has misalignment errors which will affect the accuracy of Delta measurements. In order to minimize the effect of the errors, the Hall element angles are measured and a correction is performed for their misalignment. This note discusses the calibration procedure and the corrections.

1 Introduction[1]

LCLS technical note LCLS-TN-13-4[2] presented a measurement plan for the Delta undulator. Technical note LCLS-TN-13-9[3] further discussed the Hall probe array measurements. This included a discussion of the effect of small misalignments of the Hall elements. In that note, it was assumed that the misalignments would be corrected. This note presents a plan to measure the Hall element misalignments and correct for them.

The Hall probe array for the Delta undulator consists of two probes, each probe measuring all three components of the magnetic field. The three components are B_z, along the beam axis, B_y, up, and B_x in the direction making a right handed system. The probe array is illustrated in figure 1. The probe at the end of the array in the z-direction measures on the probe axis. The other
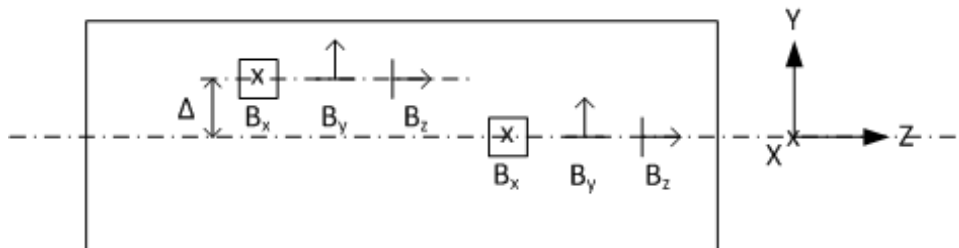

Figure 1: The Delta Hall probe array consists of two probes offset in the y-direction, each probe measuring all three components of the magnetic field.

probe measures a distance Δ offset in the y-direction. Offsets in z are calibrated and are taken into account in the analysis software. Offsets in y are also calibrated and are used in the measurements. Offsets in x are calibrated and are included in the analysis software.

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[2] Z. Wolf, "A Magnetic Measurement Plan For The Delta Undulator", LCLS-TN-13-4, March, 2013.

[3] Z. Wolf, "Hall Probe Array Measurements Of The Delta Undulator", LCLS-TN-13-9, November, 2013.

2   Measurements With Hall Element Angle Errors

Suppose a Hall element which is properly aligned is sensitive in the direction of unit vector $\widehat{n}$. The element measures $B_n = \overline{B} \cdot \widehat{n}$. We typically want $\widehat{n}$ to point in the direction of a coordinate axis so that we measure the component of the field in the axis direction.

Suppose the Hall element is misaligned. Now the element is sensitive in the direction $\widehat{m}$. We measure $B_m = \overline{B} \cdot \widehat{m}$. The direction $\widehat{m}$ is related to the desired direction $\widehat{n}$ through a rotation $\overline{\overline{R}}$, $\widehat{m} = \overline{\overline{R}} \cdot \widehat{n}$.

We can represent a vector as a column matrix of the components along fixed x, y, and z axes. The rotation operator is represented as a matrix in this coordinate system. Rotations are considered in the active sense, they move the sensitive axis of a Hall element from the desired direction to the actual direction. We build the rotation up out of three rotations: roll, pitch, and yaw. Roll is a rotation by θ_R whose axis is along $+z$ and which moves a vector from the $+x$ direction toward the $+y$ direction. Pitch is a rotation by θ_P whose axis is along $+x$ and which moves a vector from the $+z$ direction toward the $+y$ direction. Yaw is a rotation by θ_Y whose axis is along $+y$ and which moves a vector from the $+x$ direction toward the $+z$ direction. These rotations are illustrated in figure 2.
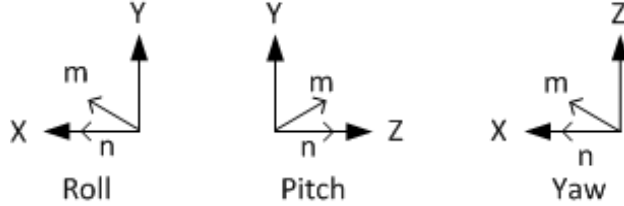

Figure 2: Roll, pitch, and yaw rotations of the Hall element sensitive direction are illustrated.

Roll is represented by the rotation matrix

$$R_R(\theta_R) = \begin{pmatrix} \cos(\theta_R) & -\sin(\theta_R) & 0 \\ \sin(\theta_R) & \cos(\theta_R) & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{1}$$

Pitch is represented by the rotation matrix

$$R_P(\theta_P) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_P) & \sin(\theta_P) \\ 0 & -\sin(\theta_P) & \cos(\theta_P) \end{pmatrix} \tag{2}$$

Yaw is represented by the rotation matrix

$$R_Y(\theta_Y) = \begin{pmatrix} \cos(\theta_Y) & 0 & -\sin(\theta_Y) \\ 0 & 1 & 0 \\ \sin(\theta_Y) & 0 & \cos(\theta_Y) \end{pmatrix} \tag{3}$$

An arbitrary rotation can be made by first rotating in roll, then pitch, and then yaw.

$$R(\theta_R, \theta_P, \theta_Y) = R_Y(\theta_Y) R_P(\theta_P) R_R(\theta_R) \tag{4}$$

The order of the rotations is important and a different order produces a different overall rotation. We use this method of specifying rotations since it is closely tied to the fixed coordinate axes. This

makes it easier to determine the rotation angles for small rotations by using calibration magnets whose field directions are along the coordinate axes. This will be demonstrated shortly. Performing the matrix multiplications, the rotation matrix is given by

$$R(\theta_R, \theta_P, \theta_Y) = \begin{pmatrix} C_Y C_R + S_Y S_P S_R & -C_Y S_R + S_Y S_P C_R & -S_Y C_P \\ C_P S_R & C_P C_R & S_P \\ S_Y C_R - C_Y S_P S_R & -S_Y S_R - C_Y S_P C_R & C_Y C_P \end{pmatrix} \tag{5}$$

where C_α represents $\cos(\theta_\alpha)$ and S_α represents $\sin(\theta_\alpha)$, and α is R, P, or Y.

In general, the rotation angles considered here are angle errors in the Hall elements and they are small. We use this to simplify the rotation matrix. We keep second order terms on the diagonal since these terms affect the main field measurement, which affects the measured K value of the undulator. The off diagonal terms tell how rotation errors mix other field components into the component we are trying to measure. Only the largest first order terms will be kept. Furthermore, we set $\cos(\theta_\alpha) = 1$ in the off diagonal terms since the error in doing this is third order. With these simplifications, the rotation matrix becomes

$$R(\theta_R, \theta_P, \theta_Y) = \begin{pmatrix} C_Y C_R & -S_R & -S_Y \\ S_R & C_P C_R & S_P \\ S_Y & -S_P & C_Y C_P \end{pmatrix} \tag{6}$$

With this rotation matrix, we calculate the measured field values from the rotated Hall elements. They are given by $B_m = \overline{B} \cdot \widehat{m}$. Since $\widehat{m} = \overline{\overline{R}} \cdot \widehat{n}$, we have $B_m = \overline{B} \cdot \overline{\overline{R}} \cdot \widehat{n}$. In matrix form, this is

$$B_m = B^T R n \tag{7}$$

We now apply this formula to each of the Hall elements.

2.1 Measured Fields

2.1.1 B_x Hall Element

Consider the Hall element which is used to measure B_x. It has angle errors Θ_{xR}, Θ_{xP}, and Θ_{xY} for roll, pitch, and yaw, respectively. For this element, the nominal sensitive direction is $n = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}^T$ in matrix form. With the angle errors, we measure

$$B_{xm} = \begin{pmatrix} B_x & B_Y & B_z \end{pmatrix} \begin{pmatrix} \cos(\Theta_{xY})\cos(\Theta_{xR}) & -\sin(\Theta_{xR}) & -\sin(\Theta_{xY}) \\ \sin(\Theta_{xR}) & \cos(\Theta_{xP})\cos(\Theta_{xR}) & \sin(\Theta_{xP}) \\ \sin(\Theta_{xY}) & -\sin(\Theta_{xP}) & \cos(\Theta_{xY})\cos(\Theta_{xP}) \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{8}$$

This gives

$$B_{xm} = \cos(\Theta_{xY})\cos(\Theta_{xR})B_x + \sin(\Theta_{xR})B_y + \sin(\Theta_{xY})B_z \tag{9}$$

2.1.2 B_y Hall Element

Consider the Hall element which is used to measure B_y. It has angle errors Θ_{yR}, Θ_{yP}, and Θ_{yY} for roll, pitch, and yaw, respectively. For this element, $n = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}^T$. With the angle errors, we measure

$$B_{ym} = \begin{pmatrix} B_x & B_y & B_z \end{pmatrix} \begin{pmatrix} \cos(\Theta_{yY})\cos(\Theta_{yR}) & -\sin(\Theta_{yR}) & -\sin(\Theta_{yY}) \\ \sin(\Theta_{yR}) & \cos(\Theta_{yP})\cos(\Theta_{yR}) & \sin(\Theta_{yP}) \\ \sin(\Theta_{yY}) & -\sin(\Theta_{yP}) & \cos(\Theta_{yY})\cos(\Theta_{yP}) \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tag{10}$$

This gives

$$B_{ym} = -\sin(\Theta_{yR})B_x + \cos(\Theta_{yP})\cos(\Theta_{yR})B_y - \sin(\Theta_{yP})B_z \tag{11}$$

3

2.1.3 B_z Hall Element

Consider the Hall element which is used to measure B_z. It has angle errors Θ_{zR}, Θ_{zP}, and Θ_{zY} for roll, pitch, and yaw, respectively. For this element, $n = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^T$. With the angle errors, we measure

$$B_{zm} = \begin{pmatrix} B_x & B_y & B_z \end{pmatrix} \begin{pmatrix} \cos(\Theta_{zY})\cos(\Theta_{zR}) & -\sin(\Theta_{zR}) & -\sin(\Theta_{zY}) \\ \sin(\Theta_{zR}) & \cos(\Theta_{zP})\cos(\Theta_{zR}) & \sin(\Theta_{zP}) \\ \sin(\Theta_{zY}) & -\sin(\Theta_{zP}) & \cos(\Theta_{zY})\cos(\Theta_{zP}) \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$
(12)

This gives

$$B_{zm} = -\sin(\Theta_{zY})B_x + \sin(\Theta_{zP})B_y + \cos(\Theta_{zY})\cos(\Theta_{zP})B_z$$
(13)

2.1.4 Summary

We can put the equations for the measured fields in matrix form to summarize the results.

$$\begin{pmatrix} B_{xm} \\ B_{ym} \\ B_{zm} \end{pmatrix} = \begin{pmatrix} \cos(\Theta_{xY})\cos(\Theta_{xR}) & \sin(\Theta_{xR}) & \sin(\Theta_{xY}) \\ -\sin(\Theta_{yR}) & \cos(\Theta_{yP})\cos(\Theta_{yR}) & -\sin(\Theta_{yP}) \\ -\sin(\Theta_{zY}) & \sin(\Theta_{zP}) & \cos(\Theta_{zY})\cos(\Theta_{zP}) \end{pmatrix} \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix}$$
(14)

where B_{xm} is the field measured by the x-sensor, B_{ym} is the field measured by the y-sensor, and B_{zm} is the field measured by the z-sensor.

2.2 Measured Fields With Rotated Probe Assembly

The Delta undulator Hall probe assembly contains two probes offset from each other in the B_y element direction. Measurements are made with the probe assembly in the standard orientation, where the offset is in the y-direction, and also in a rotated orientation, where the offset is in the x-direction. We now consider the effect of angle errors when the probe assembly is rotated.

When the probe assembly is rotated, each Hall element has its sensitivity direction rotated by 90 degrees. The rotation is an overall roll by $-90°$. The resulting element sensitivity directions are

$$\widehat{m} = \overline{\overline{R}}_R(-90°) \cdot \overline{\overline{R}} \cdot \widehat{n}$$
(15)

where $\overline{\overline{R}} \cdot \widehat{n}$ is the element sensitive direction after angle errors are applied. In matrix form, the measured fields are

$$B_m = B^T R_R(-90°) R n$$
(16)

We expand $B^T R_R(-90°)$ to get

$$B^T R_R(-90°) = \begin{pmatrix} B_x & B_y & B_z \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
(17)

$$= \begin{pmatrix} -B_y & B_x & B_z \end{pmatrix}$$
(18)

We find that the measured fields from each element have the same form as when the probe offset is vertical, but with $-B_y$ replacing B_x, and B_x replacing B_y. We find

$$B_{xm} = \cos(\Theta_{xY})\cos(\Theta_{xR})(-B_y) + \sin(\Theta_{xR})B_x + \sin(\Theta_{xY})B_z$$
(19)

$$B_{ym} = -\sin(\Theta_{yR})(-B_y) + \cos(\Theta_{yP})\cos(\Theta_{yR})B_x - \sin(\Theta_{yP})B_z$$
(20)

$$B_{zm} = -\sin(\Theta_{zY})(-B_y) + \sin(\Theta_{zP})B_x + \cos(\Theta_{zY})\cos(\Theta_{zP})B_z$$
(21)

where B_{xm} is the field measured by the x-sensor, etc.

We can put the equations for the measured fields with the probe assembly rotated in matrix form to summarize the results.

$$
\begin{pmatrix} B_{xm} \\ B_{ym} \\ B_{zm} \end{pmatrix} = \begin{pmatrix} \cos(\Theta_{xY})\cos(\Theta_{xR}) & \sin(\Theta_{xR}) & \sin(\Theta_{xY}) \\ -\sin(\Theta_{yR}) & \cos(\Theta_{yP})\cos(\Theta_{yR}) & -\sin(\Theta_{yP}) \\ -\sin(\Theta_{zY}) & \sin(\Theta_{zP}) & \cos(\Theta_{zY})\cos(\Theta_{zP}) \end{pmatrix} \begin{pmatrix} -B_y \\ B_x \\ B_z \end{pmatrix} \quad (22)
$$

where B_{xm} is the field measured by the x-sensor, B_{ym} is the field measured by the y-sensor, and B_{zm} is the field measured by the z-sensor.

3 Hall Element Angle Error Calibrations

3.1 B_x Calibration Magnet

Suppose we put the Hall probe assembly in a calibration magnet with only B_x. In this case, when the probe offset is in the y-direction, we find

$$
\begin{pmatrix} B_{xm} \\ B_{ym} \\ B_{zm} \end{pmatrix} = \begin{pmatrix} \cos(\Theta_{xY})\cos(\Theta_{xR}) \\ -\sin(\Theta_{yR}) \\ -\sin(\Theta_{zY}) \end{pmatrix} B_x \quad (23)
$$

The measured field from the y-sensor gives

$$
\Theta_{yR} = -B_{ym}/B_x \quad (24)
$$

for small angles. The measured field from the z-sensor gives

$$
\Theta_{zY} = -B_{zm}/B_x \quad (25)
$$

for small angles.

If we rotate the probe so the offset is in the x-direction, we find

$$
\begin{pmatrix} B_{xm} \\ B_{ym} \\ B_{zm} \end{pmatrix} = \begin{pmatrix} \sin(\Theta_{xR}) \\ \cos(\Theta_{yP})\cos(\Theta_{yR}) \\ \sin(\Theta_{zP}) \end{pmatrix} B_x \quad (26)
$$

The measured field from the x-sensor gives

$$
\Theta_{xR} = B_{xm}/B_x \quad (27)
$$

for small angles. The measured field from the z-sensor gives

$$
\Theta_{zP} = B_{zm}/B_x \quad (28)
$$

for small angles.

3.2 B_y Calibration Magnet

Suppose we put the Hall probe assembly in a calibration magnet with only B_y. In this case, when the probe offset is in the y-direction, we find

$$
\begin{pmatrix} B_{xm} \\ B_{ym} \\ B_{zm} \end{pmatrix} = \begin{pmatrix} \sin(\Theta_{xR}) \\ \cos(\Theta_{yP})\cos(\Theta_{yR}) \\ \sin(\Theta_{zP}) \end{pmatrix} B_y \quad (29)
$$

The measured field from the x-sensor gives

$$\Theta_{xR} = B_{xm}/B_y \tag{30}$$

for small angles. The measured field from the z-sensor gives

$$\Theta_{zP} = B_{zm}/B_y \tag{31}$$

for small angles.

If we rotate the probe so the offset is in the x-direction, we find

$$\begin{pmatrix} B_{xm} \\ B_{ym} \\ B_{zm} \end{pmatrix} = \begin{pmatrix} \cos(\Theta_{xY})\cos(\Theta_{xR}) \\ -\sin(\Theta_{yR}) \\ -\sin(\Theta_{zY}) \end{pmatrix} (-B_y) \tag{32}$$

The measured field from the y-sensor gives

$$\Theta_{yR} = B_{ym}/B_y \tag{33}$$

for small angles. The measured field from the z-sensor gives

$$\Theta_{zY} = B_{zm}/B_y \tag{34}$$

for small angles.

3.3 B_z Calibration Magnet

Suppose we put the Hall probe assembly in a calibration magnet with only B_z. In this case, when the probe offset is in the y-direction, we find

$$\begin{pmatrix} B_{xm} \\ B_{ym} \\ B_{zm} \end{pmatrix} = \begin{pmatrix} \sin(\Theta_{xY}) \\ -\sin(\Theta_{yP}) \\ \cos(\Theta_{zY})\cos(\Theta_{zP}) \end{pmatrix} B_z \tag{35}$$

The measured field from the x-sensor gives

$$\Theta_{xY} = B_{xm}/B_z \tag{36}$$

for small angles. The measured field from the y-sensor gives

$$\Theta_{yP} = -B_{ym}/B_z \tag{37}$$

for small angles.

If we rotate the probe so the offset is in the x-direction, we find

$$\begin{pmatrix} B_{xm} \\ B_{ym} \\ B_{zm} \end{pmatrix} = \begin{pmatrix} \sin(\Theta_{xY}) \\ -\sin(\Theta_{yP}) \\ \cos(\Theta_{zY})\cos(\Theta_{zP}) \end{pmatrix} B_z \tag{38}$$

The measured field from the x-sensor gives

$$\Theta_{xY} = B_{xm}/B_z \tag{39}$$

for small angles. The measured field from the y-sensor gives

$$\Theta_{yP} = -B_{ym}/B_z \tag{40}$$

for small angles. These are the same formulas as without the probe offset rotation.

3.4 Summary

The Hall element angle error calibration results are summarized in the following table:

Calibration Magnet	Probe Offset Vertical	Probe Offset Horizontal
B_x	$\Theta_{yR} = -B_{ym}/B_x$, $\Theta_{zY} = -B_{zm}/B_x$	$\Theta_{xR} = B_{xm}/B_x$, $\Theta_{zP} = B_{zm}/B_x$
B_y	$\Theta_{xR} = B_{xm}/B_y$, $\Theta_{zP} = B_{zm}/B_y$	$\Theta_{yR} = B_{ym}/B_y$, $\Theta_{zY} = B_{zm}/B_y$
B_z	$\Theta_{xY} = B_{xm}/B_z$, $\Theta_{yP} = -B_{ym}/B_z$	$\Theta_{xY} = B_{xm}/B_z$, $\Theta_{yP} = -B_{ym}/B_z$

Put another way, the following table indicates which calibration magnet measurement gives each angle error. In this table V represents that the probe offset is vertical, and H represents that the probe offset is horizontal.

Error	B_x, V	B_y, V	B_z, V	B_x, H	B_y, H	B_z, H
$\Theta_{xR} =$		B_{xm}/B_y		B_{xm}/B_x		
$\Theta_{xY} =$			B_{xm}/B_z			B_{xm}/B_z
$\Theta_{yR} =$	$-B_{ym}/B_x$				B_{ym}/B_y	
$\Theta_{yP} =$			$-B_{ym}/B_z$			$-B_{ym}/B_z$
$\Theta_{zP} =$		B_{zm}/B_y		B_{zm}/B_x		
$\Theta_{zY} =$	$-B_{zm}/B_x$				B_{zm}/B_y	

4 Hall Element Angle Error Corrections

We measure B_{xm}, B_{ym}, and B_{zm}; and we want to know B_x, B_y, and B_z. We use equation 14. It has the form $B_m = TB$. From the calibration, we know T, and from the measurements, we know B_m. We find B as $B = T^{-1}B_m$. In particular, we have

$$\begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \begin{pmatrix} \cos(\Theta_{xY})\cos(\Theta_{xR}) & \sin(\Theta_{xR}) & \sin(\Theta_{xY}) \\ -\sin(\Theta_{yR}) & \cos(\Theta_{yP})\cos(\Theta_{yR}) & -\sin(\Theta_{yP}) \\ -\sin(\Theta_{zY}) & \sin(\Theta_{zP}) & \cos(\Theta_{zY})\cos(\Theta_{zP}) \end{pmatrix}^{-1} \begin{pmatrix} B_{xm} \\ B_{ym} \\ B_{zm} \end{pmatrix} \quad (41)$$

Similarly, when the probe offset is horizontal, we use equation 22 to find

$$\begin{pmatrix} -B_y \\ B_x \\ B_z \end{pmatrix} = \begin{pmatrix} \cos(\Theta_{xY})\cos(\Theta_{xR}) & \sin(\Theta_{xR}) & \sin(\Theta_{xY}) \\ -\sin(\Theta_{yR}) & \cos(\Theta_{yP})\cos(\Theta_{yR}) & -\sin(\Theta_{yP}) \\ -\sin(\Theta_{zY}) & \sin(\Theta_{zP}) & \cos(\Theta_{zY})\cos(\Theta_{zP}) \end{pmatrix}^{-1} \begin{pmatrix} B_{xm} \\ B_{ym} \\ B_{zm} \end{pmatrix}$$
$$(42)$$

5 Accuracy Requirements On The Hall Element Angle Calibrations

In order to determine the accuracy requirements on the Hall element angle calibrations, we return to equation 14, which we reproduce below:

$$\begin{pmatrix} B_{xm} \\ B_{ym} \\ B_{zm} \end{pmatrix} = \begin{pmatrix} \cos(\Theta_{xY})\cos(\Theta_{xR}) & \sin(\Theta_{xR}) & \sin(\Theta_{xY}) \\ -\sin(\Theta_{yR}) & \cos(\Theta_{yP})\cos(\Theta_{yR}) & -\sin(\Theta_{yP}) \\ -\sin(\Theta_{zY}) & \sin(\Theta_{zP}) & \cos(\Theta_{zY})\cos(\Theta_{zP}) \end{pmatrix} \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix}$$

As noted above, this equation has the form $B_m = TB$. Suppose we make calibration errors δT. We then think the transfer matrix is $(T + \delta T)$, where T is the matrix using the true angles. We calculate B as $B_{calc} = (T + \delta T)^{-1} B_m = (T + \delta T)^{-1} TB$. For small δT, working to first order we

find $(T + \delta T)^{-1} = \left(I - T^{-1}\delta T\right) T^{-1}$. We then find $B_{calc} \simeq \left(I - T^{-1}\delta T\right) T^{-1} T B = \left(I - T^{-1}\delta T\right) B$. The error we make from a calibration error is then $\delta B = -T^{-1}\delta T\, B$. Since $T \simeq I$, to first order $\delta B = -\delta T\, B$. This says that the error we make when calculating the field from the measurements is the negative of the change in the measured field if the Hall element was rotated by the amount of the calibration error. We will use this fact to study the effect of the calibration errors. From LCLS-TN-13-7, the following errors are important for calculating undulator parameters.

First consider the $\cos(\Theta_{xY})\cos(\Theta_{xR})$ term. These angles are not present when the probe is calibrated with an NMR since the probe is rotated to maximize its signal before the NMR calibration. This term must be present in order to accurately calculate the main field strengths and to determine the K value. The angles are known from calibrating the other terms in the transfer matrix. The corrections from the small angles are second order. Small errors in the angles, at the milliradian level, will not affect the $\cos(\Theta_{xY})\cos(\Theta_{xR})$ term. These conclusions also apply to the other diagonal elements.

Consider the $\sin(\Theta_{xR})$ term. A calibration error $\delta\Theta_{xR}$ makes an error in B_{xm} equal to $\delta B_{xm} = \delta\Theta_{xR}B_y$. In planar polarization, vertical field mode, B_y is typically 10^4 times B_x. As an upper limit, we want the error on B_x to be smaller than B_x. We thus want $\delta\Theta_{xR} < 10^{-4}$.

Consider the $\sin(\Theta_{xY})$ term. A calibration error $\delta\Theta_{xY}$ makes an error in B_{xm} equal to $\delta B_{xm} = \delta\Theta_{xY}B_z$. In planar polarization, vertical field mode, B_z is typically 100 times B_x. We want the error on B_x to be smaller than B_x. We thus want $\delta\Theta_{xY} < 10^{-2}$.

Consider the $\sin(\Theta_{yR})$ term. A calibration error $\delta\Theta_{yR}$ makes an error in B_{ym} equal to $\delta B_{ym} = \delta\Theta_{yR}B_x$. In planar polarization, horizontal field mode, B_x is typically 10^4 times B_y. We want the error on B_y to be smaller than B_y. We thus want $\delta\Theta_{yR} < 10^{-4}$.

Consider the $\sin(\Theta_{yP})$ term. A calibration error $\delta\Theta_{yP}$ makes an error in B_{ym} equal to $\delta B_{ym} = \delta\Theta_{yP}B_z$. In planar polarization, horizontal field mode, B_z is typically 100 times B_y. We want the error on B_y to be smaller than B_y. We thus want $\delta\Theta_{yP} < 10^{-2}$.

Consider the $\sin(\Theta_{zY})$ term. A calibration error $\delta\Theta_{zY}$ makes an error in B_{zm} equal to $\delta B_{zm} = \delta\Theta_{zY}B_x$. In the circular polarization modes, B_x is typically 100 times B_z. We want the error on B_z to be smaller than B_z. We thus want $\delta\Theta_{zY} < 10^{-2}$.

Consider the $\sin(\Theta_{zP})$ term. A calibration error $\delta\Theta_{zP}$ makes an error in B_{zm} equal to $\delta B_{zm} = \delta\Theta_{zP}B_y$. In the circular polarization modes, B_y is typically 100 times B_z. We want the error on B_z to be smaller than B_z. We thus want $\delta\Theta_{zP} < 10^{-2}$.

We summarize these results in the following table. We replace the "$< 10^{-2}$" with a maximum value of 0.002. We replace the "$< 10^{-4}$" with a maximum value of 5×10^{-5} in order to set a limit which is achievable.

Calibration Error	Maximum Value (mrad)
$\delta\Theta_{xR}$	0.05
$\delta\Theta_{xY}$	2
$\delta\Theta_{yR}$	0.05
$\delta\Theta_{yP}$	2
$\delta\Theta_{zY}$	2
$\delta\Theta_{zP}$	2

6 Conclusion

This note showed the effect of Hall element angles on the Hall probe measurements of the Delta undulator. A prescription to measure the Hall element angles using calibration magnets was presented. Also presented was the equation to calculate the true field given the measured field and the Hall element angles. Limits on the errors in the calibration of the Hall element angles were presented.

Acknowledgements