Effect Of Quadrant Bow On Delta Undulator Phase Errors

Zachary Wolf
SLAC

February 18, 2015

Abstract

The Delta undulator quadrants are tuned individually and are then assembled to make the tuned undulator. Bow of the quadrants on the tuning stand can cause large phase errors in the assembled undulator. The effect of quadrant bow is studied in this note and limits on bow are derived in order to limit phase errors.

1 Introduction[1]

The Delta undulator[2] is tuned by individually tuning each of the four quadrants. The permanent magnet material of the quadrants has relative permeability approximately equal to 1, so the magnetic field of the assembled undulator is approximately the same as the superposition of the fields from the four quadrants. If each quadrant is tuned to give straight trajectories and small phase errors, the assembled undulator should give straight trajectories and small phase errors to good approximation.

The method of tuning each quadrant individually is very sensitive to the position of the Hall probe relative to the quadrant. The magnetic field from a quadrant decays exponentially with distance away from the quadrant. If the quadrants are systematically bent during tuning, phase errors are introduced in the assembled undulator as explained below. Similarly, if the undulator is assembled with the quadrants bent, phase errors are introduced.

When tuning the Delta undulator, it was noticed that the weight of the quadrants introduced a systematic bow in their shape while they were being tuned. In this note, we detail the effect of this bow, and we set limits on the bow in order to limit the resulting phase errors.

2 Quadrant Bow During Tuning

During the tuning of the Delta undulator, the quadrants were held in a kinematic mount and their weight caused a vertical bow in their shape. Figure 1 shows CMM measurements of the bow of quadrant 2. The size of the bow is approximately 30 microns.

The bow during tuning leads to field errors in the assembled undulator as outlined in figure 2. When the quadrant is placed on the bench, it assumes the bowed shape shown in part A) of the figure. The measurement probe follows a straight line over the magnets and we assume it is at the beam axis location in the center of the quadrant. The beam axis is indicated by the z-axis in the figure. The quadrant is tuned by "virtual shimming", that is, by moving the permanent magnets. Effectively, the magnets are moved so that they follow a straight line despite the curvature in the

[1] Work supported in part by the DOE Contract DE-AC02-76SF00515. This work was performed in support of the LCLS project at SLAC.
[2] A. Temnykh, Physical Review Special Topics-Accelerators and Beams **11**, 120702 (2008).

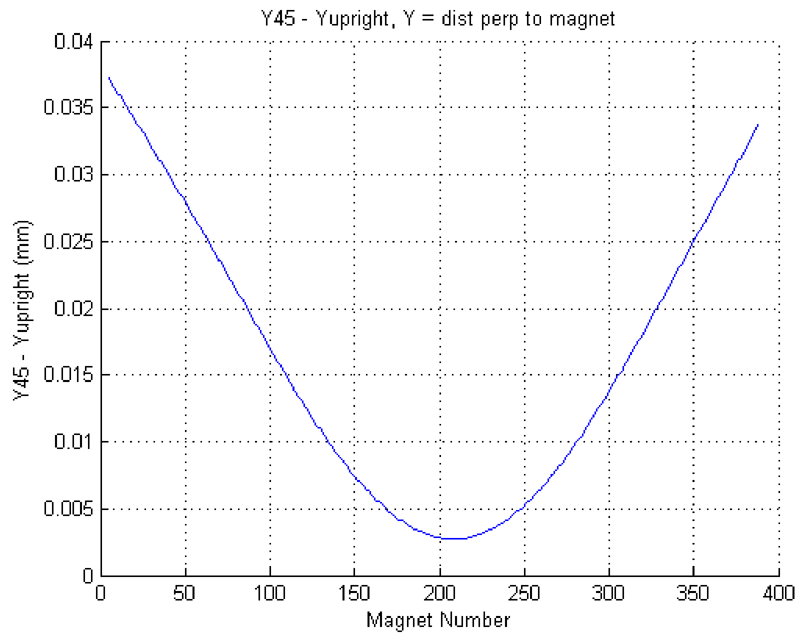

Figure 1: When the quadrants were placed on the bench, they bowed by approximately 30 microns.

magnet keeper. This is shown in part B) of the figure. When the keeper is straightened in the assembled undulator, the magnets move relative to the beam axis and effectively reverse their original curvature as shown in part C) of the figure. The magnets are further from the beam axis at the ends of the undulator and the fields are weaker there. The change in field strength along the undulator causes phase errors, as discussed below.

3 Phase Errors In The Assembled Undulator

Suppose the magnet arrays in the assembled undulator assume the inverse of the bow of the keeper when the quadrant was placed on the measurement bench, as discussed in the previous section. We derive below the phase errors resulting from such a bow in all four quadrants. We consider the Delta undulator in planar, vertical field mode, but the results are easily extended to other polarization modes.

The slippage between two points a and b in a planar undulator with vertical magnetic field is given by[3]

$$S_{ab} = \int_a^b \frac{1}{2\gamma^2} + \frac{1}{2}x'^2 dz \tag{1}$$

where γ is the Lorentz factor and x' is the slope of the horizontal trajectory in the vertical undulator field. The horizontal trajectory slope is given by

$$x' = \frac{\widetilde{K}}{\gamma}\cos(k_u z) \tag{2}$$

where $\widetilde{K}$ is the local undulator parameter at the z-position of interest, and $k_u = 2\pi/\lambda_u$, where λ_u is the undulator period. If we consider two points a and b spaced apart by one period, the slippage

[3] Z. Wolf, "Introduction To LCLS Undulator Tuning", LCLS-TN-04-7, June, 2004.

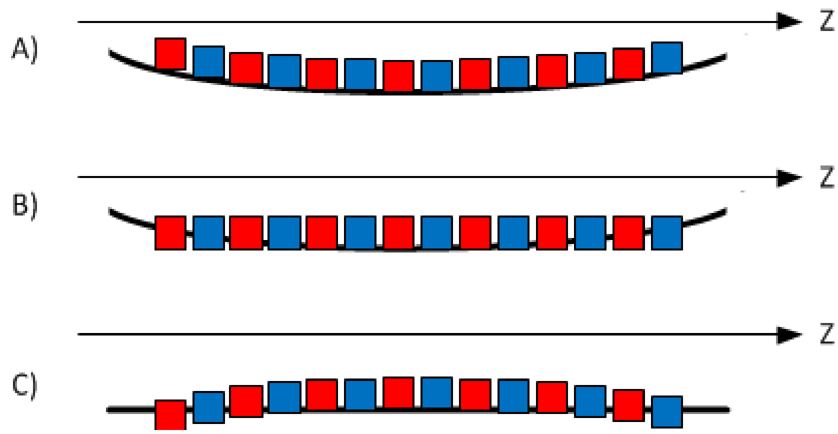

Figure 2: A) The quadrants assume a bowed shape when placed on the bench. B) Tuning the quadrant effectively moves the magnets to a straight line, even though the magnet keeper is bowed. C) When the quadrant structures are placed in the undulator, the keepers assume a straight shape, but the magnet positions have a bow.

in the period is given by

$$S_{\lambda_u} = \frac{1}{2\gamma^2}\left(1 + \frac{1}{2}\widetilde{K}^2\right)\lambda_u \tag{3}$$

The vertical magnetic field can be considered as an ideal field B_y plus small errors ΔB_y that change with position. We assume the magnetic field keeps its sinusoidal z-dependence to good approximation and the small errors make field changes over long distances compared to a period. The local undulator parameter $\widetilde{K}$ can be written as an ideal value K, plus a small position dependent change ΔK.

$$\widetilde{K} = K + \Delta K \tag{4}$$

The error in the field causes a change in the slippage per period compared to the ideal case of

$$\Delta S_{\lambda_u} = \frac{\lambda_u}{2\gamma^2} K \Delta K \tag{5}$$

Since the K value is proportional to the field, we have

$$\frac{\Delta K}{K} = \frac{\Delta B_y}{B_y} \tag{6}$$

So

$$\Delta S_{\lambda_u} = \frac{\lambda_u}{2\gamma^2} K^2 \frac{\Delta B_y}{B_y}$$

The change in the phase corresponding to the slippage change is given by

$$\Delta P = \frac{2\pi}{\lambda_r} \Delta S \tag{7}$$

where, to good approximation, we can use the radiation wavelength from the ideal undulator, which is given by

$$\lambda_r = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{1}{2}K^2\right) \tag{8}$$

So the change in the phase per period compared to the ideal case is

$$\Delta P_{\lambda_u} = 2\pi \frac{K^2}{(1+\frac{1}{2}K^2)} \frac{\Delta B_y}{B_y} \tag{9}$$

The fields from a Delta undulator quadrant are given by[4]

$$B_r = B_{0q} \exp(-k_u r) \cos(k_u(z - z_0)) \tag{10}$$

where r is measured away from the quadrant and z is measured along the quadrant. If the quadrant bows upward by $+\Delta y_q$ during tuning when it is placed on the bench, the distance from the magnets to the beam axis changes by $\Delta r = -\Delta y_q$. The field on the beam axis changes by

$$\frac{\Delta B_r}{B_r} = +k_u \Delta y_q \tag{11}$$

Positive position changes of the quadrant make the magnets move closer to the beam axis, increasing the field from the quadrant. The undulator will be tuned to take out this field change. When the quadrant is assembled straight, the negative of this field change will appear in the assembled undulator.

$$\left(\frac{\Delta B_r}{B_r}\right)_{undulator} = -k_u \Delta y_q \tag{12}$$

When the assembled undulator is in planar, vertical field mode, the vertical undulator field is given by

$$B_y = \frac{4B_r}{\sqrt{2}} \tag{13}$$

so

$$\left(\frac{\Delta B_y}{B_y}\right)_{undulator} = \left(\frac{\Delta B_r}{B_r}\right)_{undulator} = -k_u \Delta y_q \tag{14}$$

assuming the bend in the quadrants is the same for all quadrants.

The change in the slippage per period in the assembled undulator compared to the case when the quadrants are tuned straight is then

$$\Delta S_{\lambda_u} = -\frac{\lambda_u}{2\gamma^2} K^2 k_u \Delta y_q \tag{15}$$

and the change in phase per period is

$$\Delta P_{\lambda_u} = -2\pi \frac{K^2}{(1+\frac{1}{2}K^2)} k_u \Delta y_q \tag{16}$$

For the Delta undulator, the period is 0.032 m. Inserting a value of $K = 3$ and a position change of $\Delta y_q = 10$ microns gives a phase change of -1.16 degree per period. This difference will accumulate as one moves down the undulator.

Suppose the undulator has length L and we take $z = 0$ at the undulator center. The accumulated slippage change along the undulator from initial position $z = -L/2$ where the slippage difference is zero to an arbitrary z is given by

$$\Delta S(z) = \int_{-L/2}^{z} -\frac{\lambda_u}{2\gamma^2} K^2 k_u \Delta y_q(z') \frac{dz'}{\lambda_u} \tag{17}$$

The quadrants are supported at two points and bend on the test bench in an approximately quadratic manner. We take the position to be correct at the center, which means that the Hall probe is at

[4] Z. Wolf, "A Calculation Of The Fields In The Delta Undulator", LCLS-TN-14-1, January, 2014.

the correct beam axis location at the center of the quadrant. Suppose the quadrant bends up by $\Delta y_{q\,\max}$ at the ends. With the approximate quadratic bending, the bend in the quadrant is given by

$$\Delta y_q(z') = \Delta y_{q\,\max}\left(\frac{z'}{L/2}\right)^2 \tag{18}$$

From the undulator entrance to an arbitrary location z, the bending of the quadrants during tuning causes a slippage change in the undulator compared to the unbent case of

$$\Delta S(z) = -\frac{\lambda_u}{2\gamma^2}K^2\frac{k_u}{\lambda_u}\Delta y_{q\,\max}\frac{4}{3L^2}\left(z^3 + \left(\frac{L}{2}\right)^3\right) \tag{19}$$

and the corresponding change in phase is

$$\Delta P(z) = -2\pi\frac{K^2}{(1+\frac{1}{2}K^2)}\frac{k_u}{\lambda_u}\Delta y_{q\,\max}\frac{4}{3L^2}\left(z^3 + \left(\frac{L}{2}\right)^3\right) \tag{20}$$

Without errors, the undulator has constant K value along its length. The slippage as a function of z is given by

$$S_0(z) = \frac{1}{2\gamma^2}\left(1+\frac{1}{2}K^2\right)z \tag{21}$$

With the errors, the slippage as a function of z becomes $S = S_0 + \Delta S$:

$$S(z) = \frac{1}{2\gamma^2}\left(1+\frac{1}{2}K^2\right)z - \frac{\lambda_u}{2\gamma^2}K^2\frac{k_u}{\lambda_u}\Delta y_{q\,\max}\frac{4}{3L^2}\left(z^3 + \left(\frac{L}{2}\right)^3\right) \tag{22}$$

and the phase as a function of z is

$$P(z) = \frac{2\pi}{\lambda_u}z - 2\pi\frac{K^2}{(1+\frac{1}{2}K^2)}\frac{k_u}{\lambda_u}\Delta y_{q\,\max}\frac{4}{3L^2}\left(z^3 + \left(\frac{L}{2}\right)^3\right) \tag{23}$$

In order to determine an effective K value for the undulator, and also the rms phase error in the undulator, a linear fit is made to the slippage as a function of z. The linear fit has slope M and the slope gives the effective K value, K_{eff}, as follows:

$$M = \frac{1}{2\gamma^2}\left(1+\frac{1}{2}K_{eff}^2\right) \tag{24}$$

and

$$K_{eff} = \sqrt{2\left(2\gamma^2 M - 1\right)} \tag{25}$$

The residuals of the fit give the phase errors.

$$\epsilon_\phi = \frac{2\pi}{\lambda_r}\times residuals \tag{26}$$

where in this case we use the measured effective K value to determine the radiation wavelength

$$\lambda_r = \frac{\lambda_u}{2\gamma^2}\left(1+\frac{1}{2}K_{eff}^2\right) \tag{27}$$

The Delta undulator has $L = 96\lambda_u$ and $\lambda_u = .032$ m. Take $K = 3$, and $\gamma = 10^4$ although the value of γ drops out of the phase calculations. We take $\Delta y_{q\,\max} = 30$ microns, which is the measured value when the Delta quadrants were initially tuned. The phase in the undulator both

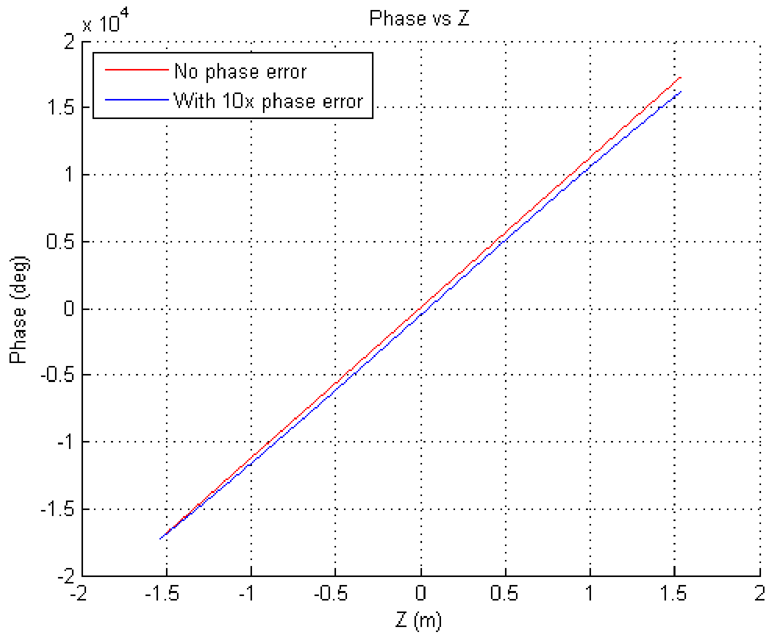

Figure 3: Phase in the undulator both without errors and with errors scaled by a factor of 10.

without errors and with scaled errors multiplied by a factor of 10 for illustration is shown in figure 3. As you can see, the phase change from the quadrant bow is small on the scale of the total phase change, but it is visible at the undulator exit. The change in phase is shown by itself in figure 4. These are the actual changes and the previous factor of 10 scaling is not included here or from this point on. When a linear fit is made to the slippage as a function of z, the K value shifts from its ideal value of 3 to a value of $K_{eff} = 2.9965$, and the residuals of the fit give the phase errors. The phase errors are shown in figure 5. The rms phase error is 8.4 degrees. For small bow, the rms phase error scales linearly with the size of the quadrant bow during tuning, with slope 2.8 degrees per 10 μm of bow. If we wish to limit the rms phase errors from quadrant bow to 5 degrees, the quadrants must be tuned with a bow of less than 18 μm.

4 Conclusion

If the quadrants are tuned with a bow due to their weight, it produces a systematic offset of the magnet locations in the assembled undulator at the ends of the undulator. This produces a phase error with a cubic z-dependence. The rms phase error is 8.4 degrees for the 30 μm of bow observed in the quadrants in their kinematic mounts. If we wish to limit the rms phase error to 5 degrees, the bow must be kept below 18 μm. Similarly, systematic bow of the quadrants in the assembled undulator must be kept below 18 μm.

Acknowledgements
I am grateful to Heinz-Dieter Nuhn and Yurii Levashov for many discussions about this work.

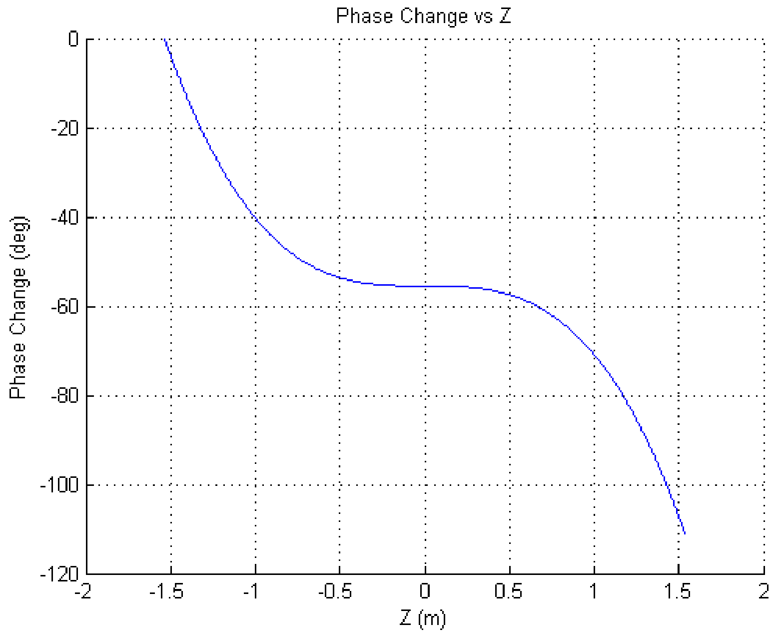

Figure 4: Change in phase from the quadrant bow.

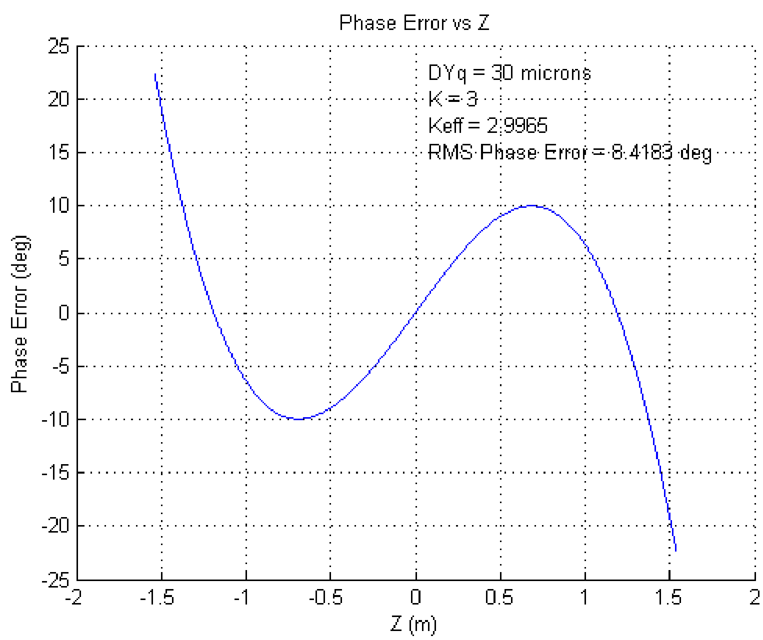

Figure 5: Phase error and K shift when the quadrants are tuned with 30 μm of bow.