ALGORITHM FOR BEAM BASED ALIGNMENT

*K. Endo, H. Fukuma and F. Q. Zhang**
National Laboratory for High Energy Physics (KEK)
** The Graduate University for Advanced Studies*
1-1 Oho, Tsukuba-shi 305, Japan

By partially reducing the small current from or adding he small current to the operational quadrupole current, the orbit of the circulating beam receives the gradient perturbation and changes a little bit from the previous orbit, if the beam orbit has an offset from the magnet center. As the effect on the orbit depends on the beam deviation from the magnet center, the magnet center offsets can be obtained if the orbit is measured whenever a part of current of each quadrupole magnet is shared with the current by-pass shunt. This procedure gives the algorithm of the beam based alignment.

A resistive shunt or transistor shunt across the magnet coil terminals gives the current reduction to the magnet which is determined by the resistance of both shunt and magnet or the terminal voltage. Assuming this scheme a preliminary test of shunt is given.

1. INTRODUCTION

In a modern synchrotron, a great number of magnets are aligned precisely along the pre-determined orbit with an accuracy of 0.1 mm (rms). To attain this accuracy, however, an indescribable effort is required. It includes booking data obtained by measurements down to a unit of μm and subsequent data processing to calculate the magnet displacement which is corrected by using tools and indicators such as micrometer to measure the relative displacements to every directions. Moreover, it requires frequent calibration of instruments. In general if surveying instruments devices are more sophisticated, they become heavier to carry them all along the accelerator ring.

So far the precise optical instruments such as level and theodolite of the first class, distance meter such as Distinvar and Distometer, tilt meter and the offset measuring device were used. And more than two electronic theodolites compose the 3D coordinate determination system with a on-line computer and a scale bar. With the help of these instruments magnets could be aligned within a tolerance. Recently the 3D system is being taken over by the laser tracking system (laser tracker) which utilizes the built-in laser interferometer. It has contributed to a significant reduction of the surveying man power, however, the accuracy obtained with the latest technology is almost same as obtained with the first class optical instruments.

Alongside of the development of the 3D coordinate determination system, a new alignment method has been devised. It is called a beam based alignment (BBA) method. Even if this method could provide us with more precise magnet positioning, the conventional method mentioned above is not dispensable because the BBA method requires the beam.

2. EQUATION OF PARTICLE MOTION

The BBA method imposes a kind of perturbation to a quadrupole magnets such as to reduce the excitation current bypassing a part of current to a shunt attached across the magnet current terminals. If there is an orbit excursion at this magnet, beam senses the change of the deflection strength and travels along the subsequent different closed orbit. The equation of motion of the particle is

$$\frac{d^2\eta}{d\phi^2}+\nu^2\eta=\nu^2\beta^{3/2}F(s),$$

where F(s)=ΔB/B p. Here Bp is the magnetic rigidity and ΔB(=B_Z-B_0) the deviation from the ideal field. The periodic solution is

$$\eta(\phi)=\frac{\nu}{2\sin\pi\nu}\int_{\phi}^{\phi+2\pi}f(\psi)\cos\nu(\pi+\phi-\psi)d\psi$$

where $f(\psi)=\beta^{3/2}F(s)$ [1]. If $\eta=\beta^{-1/2}y$, $\nu\beta d\phi=ds$ and $ds=\beta d\psi$ are introduced, the beam position at a beam monitor, placed at an azimuthal position s, is

$$y(s)=\frac{\sqrt{\beta(s)}}{2\sin\pi\nu}\sum_j\theta_j\sqrt{\beta_j}\cos\nu[\varphi(s)-\varphi_j+\pi].$$

before applying a current shunt, where $\theta_j=\Delta B\ell_j/B\rho$ is a kick angle and ℓ_j is the j-th magnet length. If a current shunt is active for the n-th quadrupole, the tune changes from v to v' and the betatron function changes,

$$y'(s)=\frac{\sqrt{\beta'(s)}}{2\sin\pi\nu'}\sum_j\theta'_j\sqrt{\beta'_j}\cos\nu'[\varphi'(s)-\varphi'_j+\pi]$$

where notations with a prime give those while the shunt is active. If $j\neq n$, then $\theta'_j=\theta_j$. Otherwise, $\theta'_n\neq\theta_n$. Then,

$$y'(s)=\frac{\sqrt{\beta'(s)}}{2\sin\pi\nu'}\begin{bmatrix}\sum_{j\neq n}\theta_j\sqrt{\beta'_j}\cos\nu'[\varphi'(s)-\varphi'_j+\pi]\\+\theta'_n\sqrt{\beta'_n}\cos\nu'[\varphi'(s)-\varphi'_n+\pi]\end{bmatrix}$$

If the tune shift due to the current shunt is small enough, $\nu'\cong\nu$ and $\beta'\cong\beta$. Then, the orbit excursion will be

$$\Delta y(s)\equiv y'(s)-y(s)=\frac{\sqrt{\beta(s)}}{2\sin\pi\nu}(\theta'_n-\theta_n)\sqrt{\beta_n}\cos\nu[\varphi(s)-\varphi_n+\pi]$$

and

$$\theta'_n-\theta_n=\xi_n\kappa G_n\ell_n/100B\rho$$

where ξ_n is the transverse displacement of the n-th quadrupole magnet and k is a percent change of the field gradient of the n-th quadrupole magnet.

$$\Delta y(s)=\frac{\sqrt{\beta(s)}}{2\sin\pi\nu}\frac{\xi_n\kappa G_n\ell_n}{100B\rho}\sqrt{\beta_n}\cos\nu[\varphi(s)-\varphi_n+\pi]$$

The observations of the beam orbit are usually done at the discrete positions. If their positions are denoted as m,

$$\Delta y_m=\frac{\sqrt{\beta_m}}{2\sin\pi\nu}\frac{\xi_n\kappa G_n\ell_n}{100B\rho}\sqrt{\beta_n}\cos\nu[\varphi_m-\varphi_n+\pi].$$

Big beam excursion is observed at the i-th position for larger beta-function. This relation is expressed in the form as

$$\Delta y_i=A_{ij}\xi_j$$

where

$$A_{ij}=\frac{\sqrt{\beta_i}}{2\sin\pi\nu}\frac{\kappa G_j\ell_j}{100B\rho}\sqrt{\beta_j}\cos\nu[\varphi_i-\varphi_j+\pi].$$

As the coefficient A_{ij} can be obtained from the linear beam optics, measurement of Δy_i directly gives the information of the quadrupole misalignment. If κ is different from magnet to magnet, A_{ij} should be modified according to the κ value.

If the gradient error given by the current shunt contributes to the betatron frequency, the tune shift in the first order approximation is given by

$$\Delta\nu = \frac{\Delta\mu}{2\pi} = \frac{1}{4\pi}\int_0^C \beta(s)\frac{\kappa K(s)}{100}ds = \frac{\kappa\beta_n K_n \ell_n}{400\pi}$$

The estimated tune shift is $\Delta\nu \approx 0.01$ for k = 5% assuming the parameters of TRISTAN-AR ring which has ν ~ 9.25. Neglecting the higher order terms and defining $\Phi(s;j)$ as

$$\Phi(s;j) = \varphi(s) - \varphi_j + \pi,$$

then,

$$y'(s) \cong \frac{\sqrt{\beta(s)}}{2\sin\pi\nu + 2\pi\Delta\nu\cos\pi\nu}\begin{bmatrix}\sum_{j\neq n}\theta_j\sqrt{\beta_j}[\cos\nu\Phi(s;j) - \Delta\nu\Phi(s;j)\sin\nu\Phi(s;j)] \\ +\theta'_n\sqrt{\beta_n}[\cos\nu\Phi(s;n) - \Delta\nu\Phi(s;n)\sin\nu\Phi(s;n)]\end{bmatrix}$$

$$\cong \frac{\sqrt{\beta(s)}}{2\sin\pi\nu}\begin{bmatrix}\sum_{j\neq n}\theta_j\sqrt{\beta_j}\{\cos[\nu\Phi(s;j)] - \Delta\nu\Phi(s;j)\sin\nu\Phi(s;j) \\ -\pi\Delta\nu\cos\nu\Phi(s;j)/\tan\pi\nu\} \\ +\theta'_n\sqrt{\beta_n}\{\cos\nu\Phi(s;n) - \Delta\nu\Phi(s;n)\sin\nu\Phi(s;n) \\ -\pi\Delta\nu\cos\nu\Phi(s;n)/\tan\pi\nu\}\end{bmatrix}$$

Taking the difference,

$$\Delta y_m = \frac{\sqrt{\beta_m}}{2\sin\pi\nu}\begin{bmatrix}\sum_{j\neq n}\theta_j\sqrt{\beta_j}\{-\Delta\nu\Phi(m;j)\sin\nu\Phi(m;j) - \pi\Delta\nu\cos\nu\Phi(m;j)/\tan\pi\nu\} \\ +\sqrt{\beta_n}\{(\theta'_n - \theta_n)\cos\nu\Phi(m;j) - \theta'_n\Delta\nu\Phi(m;n)\sin\nu\Phi(m;n) \\ -\theta'_n\pi\Delta\nu\cos\nu\Phi(m;n)/\tan\pi\nu\}\end{bmatrix}$$

where $\Phi(m;j) = \varphi_m - \varphi_j + \pi$. Then,

$$\Delta y_m = \frac{\sqrt{\beta_m}}{2\sin\pi\nu}\begin{bmatrix}-\Delta\nu\sum_j\theta_j\sqrt{\beta_j}\left\{\Phi(m;j)\sin\nu\Phi(m;j) + \frac{\pi}{\tan\pi\nu}\cos\nu\Phi(m;j)\right\} \\ +\sqrt{\beta_n}\frac{\xi_n\kappa G_n\ell_n}{100B\rho}\cos\nu\Phi(m;n)\end{bmatrix}$$

If assuming $\Delta\nu = 0$ in above equation, the former equation is obtained. The next problem is to take the difference of beta-function and phase advance into consideration.

$$\Delta y(s) \cong \frac{\sqrt{\beta'(s)}}{2\sin\pi\nu}(1 - \frac{\pi\Delta\nu}{\tan\pi\nu})\begin{bmatrix}\sum_{j\neq n}\theta_j\sqrt{\beta'_j}\cos\nu'\Phi'(s;j) \\ +\theta'_n\sqrt{\beta'_n}\cos\nu'\Phi'(s;n)\end{bmatrix} - \frac{\sqrt{\beta(s)}}{2\sin\pi\nu}\begin{bmatrix}\sum_{j\neq n}\theta_j\sqrt{\beta_j}\cos\nu\Phi(s;j) \\ +\theta_n\sqrt{\beta_n}\cos\nu\Phi(s;n)\end{bmatrix}$$

$$= \frac{1}{2\sin\pi\nu}\left[\begin{array}{l} \sum_{j\neq n}\theta_j\{\sqrt{\beta'(s)\beta_j'}\cos\nu'\Phi'(s;j) - \sqrt{\beta(s)\beta_j}\cos\nu\Phi(s;j)\} \\ +\{\theta_n'\sqrt{\beta'(s)\beta_n'}\cos\nu'\Phi'(s;n) - \theta_n\sqrt{\beta(s)\beta_n}\cos\nu\Phi(s;n)\} \\ -\frac{\pi\Delta\nu}{\tan\pi\nu}\left\{\sum_{j\neq n}\theta_j\sqrt{\beta'(s)\beta_j'}\cos\nu'\Phi'(s;j) + \theta_n'\sqrt{\beta'(s)\beta_n'}\cos\nu'\Phi'(s;n)\right\} \end{array}\right]$$

According to the first order approximation,

$$\Delta y_m \cong \frac{\sqrt{\beta_m}}{2\sin\pi\nu}\left[\sum_j\theta_j\sqrt{\beta_j}\left\{\begin{array}{l}[\frac{\Delta\beta_m}{2\beta_m} + \frac{\Delta\beta_j}{2\beta_j} - \frac{\pi\Delta\nu}{\tan\pi\nu}]\cos\nu\Phi_{mj} \\ -[\Phi_{mj}\Delta\nu + \nu\Delta\Phi_{mj}]\sin\nu\Phi_{mj}\end{array}\right\} + \Delta\theta_n\sqrt{\beta_n}\cos\nu\Phi_{mn}\right]$$

where $\Phi_{mj} \equiv \Phi(m;j)$, $\nu' = \nu + \Delta\nu$, $\beta_j' = \beta_j + \Delta\beta_j$, $\Phi'(m;j) = \Phi(m;j) + \Delta\Phi(m;j)$ and $\theta_n' = \theta_n + \Delta\theta_n$. Defining $\{\Delta y\}$ and $\{\xi\}$ as a beam position displacement column vector and a magnet offset column vector of either horizontal or vertical plane, respectively,

$$\{\Delta y\} = (B)\{\xi\}$$

where B is a NxN matrix whose elements are

$$B_{mj} = \frac{\sqrt{\beta_m\beta_j}}{2\sin\pi\nu}\frac{G_j\ell_j}{B\rho}\left\{\begin{array}{l}[\frac{\Delta\beta_m}{2\beta_m} + \frac{\Delta\beta_j}{2\beta_j} - \frac{\pi\Delta\nu}{\tan\pi\nu}]\cos\nu\Phi_{mj} \\ -[\Phi_{mj}\Delta\nu + \nu\Delta\Phi_{mj}]\sin\nu\Phi_{mj}\end{array}\right\} \text{ for } j\neq n, \text{ and}$$

$$B_{mn} = \frac{\sqrt{\beta_m\beta_n}}{2\sin\pi\nu}\frac{G_n\ell_n}{B\rho}\left\{\begin{array}{l}[\frac{\Delta\beta_m}{2\beta_m} + \frac{\Delta\beta_n}{2\beta_n} - \frac{\pi\Delta\nu}{\tan\pi\nu} + \frac{\kappa}{100}]\cos\nu\Phi_{mn} \\ -[\Phi_{mn}\Delta\nu + \nu\Delta\Phi_{mn}]\sin\nu\Phi_{mn}\end{array}\right\} \text{ for } j = n$$

$$m, j = 1,2,3,\ldots\ldots,N.$$

If the matrix elements, B_{mj}, are given numerically for a fixed n, all magnet transverse displacements can be given by

$$\{\xi\} = (B)^{-1}\{\Delta y\}.$$

This relation is obtained for both horizontal and vertical plane independently when no coupling exists between both planes. As the effect of the current shunt on one quadrupole appears in all BPM, one BPM may be used to monitor the change of the beam orbit.

Even if κ=0, misalignments can be found by solving above matrix equation. However, this solution will be less accurate because the BPM errors cannot be excluded [2].

3. NUMERICAL SIMULATION FOR TRISTAN-AR RING

To get the generalized relation, the (B) matrix must be estimated numerically. The TRISTAN-AR ring is selected for this purpose. Two optics, with and without a current shunt to one specified quadrupole magnet, are calculated using a MAGIC like optics code which has been developed for this purpose. If the current shunt allows the current reduction by 5% for QR8-NW quadrupole, every beam position monitor gives different reading. Two typical readings are shown in Fig. 1, where BPM's close to QC4-SW and QF10-NW are selected. (QC4 and QF10 are the quad names. SW and NW give their location.) Beam position changes linearly with the magnitude of misalignment.

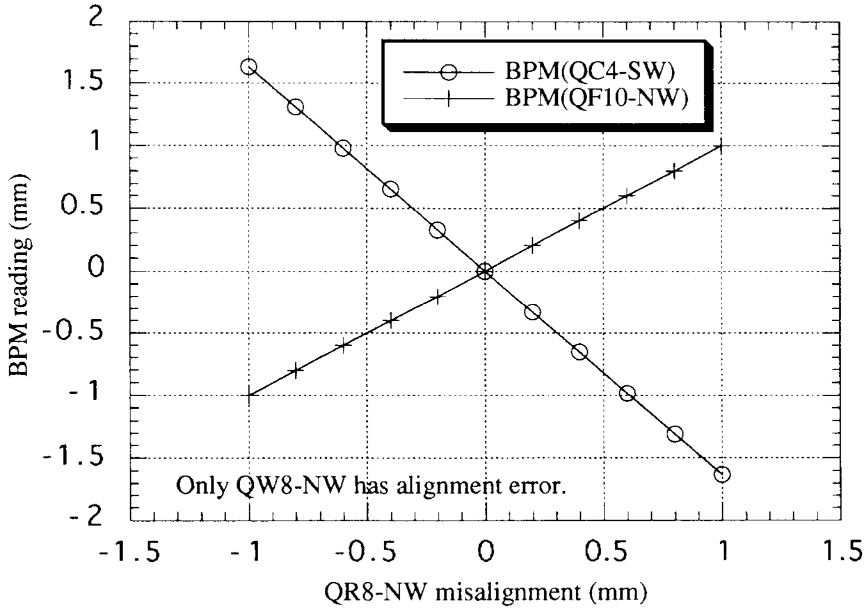

Fig. 1 Estimated beam position monitor readings for $\kappa = 5\%$ for misalignment of a quadrupole magnet (QR8-NW) at two points, 1300 deg. upstream (QC4-SW) and 270 deg. downstream (QF10-NW) of QR8-NW.

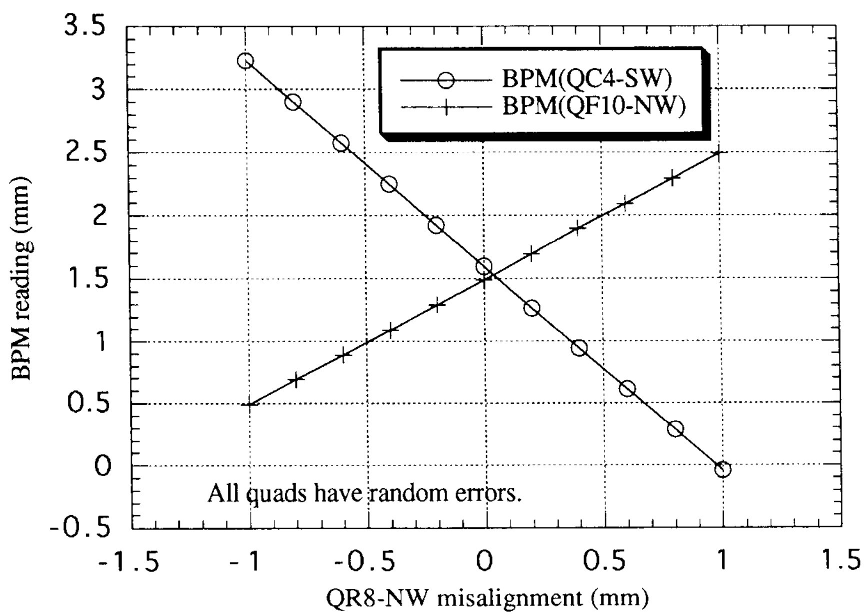

Fig.2 Estimated beam position monitor readings for $\kappa = 5\%$ for misalignment of a quadrupole magnet (QR8-NW) at two points, 1300 deg. upstream (QC4SW) and 270 deg. downstream (QF10-NW) of QR8-NW. All quads have random alignment errors in addition to the fixed error of QR8-NW quad.

If the linear relations at a specified BPM can be established for all quadrupole magnets, misalignments can be predicted by activating the current shunt sequentially. Every quadrupole has alignment errors more or less even if the orbit error is corrected with steering dipoles, however, their effects are piled up and the BPM readings change retaining a linear relation as shown in Fig.2 where are assumed random misalignments to all quadrupoles except for QR8-NW which has a fixed misalignment.

If the orbit is not corrected with steering dipoles, BPM readings reflect all quadrupole misalignments. Taking the orbit difference before and after the current shunt is activated, the related quadrupole misalignment can be obtained.

4. PRELIMINARY RESULTS WITH MODEL SHUNT CIRCUITS

A current shunt is a circuit connected parallel to a quadrupole magnet as shown in Fig.3. A part of the exciting current of the quadrupole flows through this circuit while it is conducting. It modifies the quadrupole current and affects the beam orbit if the closed orbit deviation from an ideal one is not zero. Two kinds of shunt is realized, resistive and FET current shunts.

The resistive shunt uses the fixed resistor parallel to the magnet coil and the current sharing between the shunt and the magnet balances when the both voltages become same [3]. If the magnet is excited stationary, its condition is $V=I_m R_m = I_s R_s$, where I and R are the current and resistivity, respectively, and suffices m and s mean the magnet and shunt, respectively. In the present case when it is applied to the TRISTAN-AR magnet, R_s=0.2, 0.4 and 0.8Ω, and R_m=0.006Ω. Assuming I_m=500A, I_s=14.56A (2.9%), 7.38A (1.5%) and 3.72A (0.74%), respectively. Fig.4 gives the typical current wave form shared to the resistive shunt. Top trace is the voltage of the shunt resistor and bottom trace is the output of an isolation amplifier for the current monitoring.

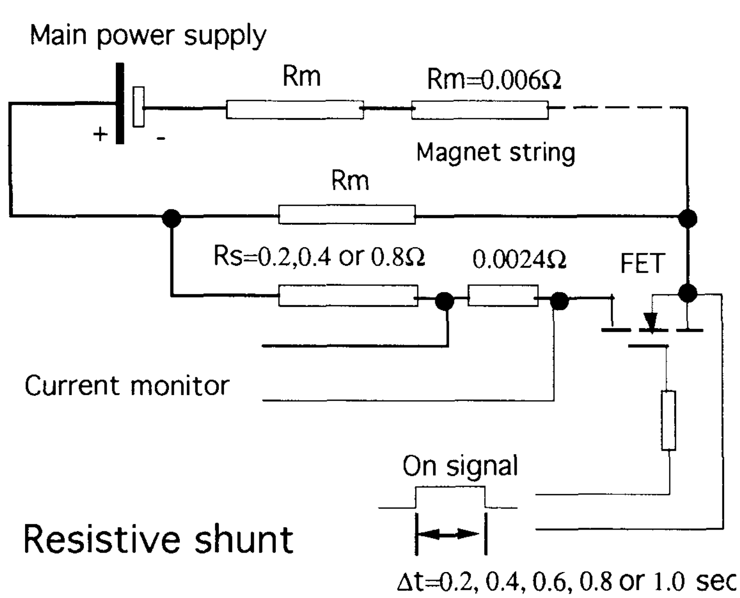

Fig. 3 Simplified model circuit of the resistive current shunt.

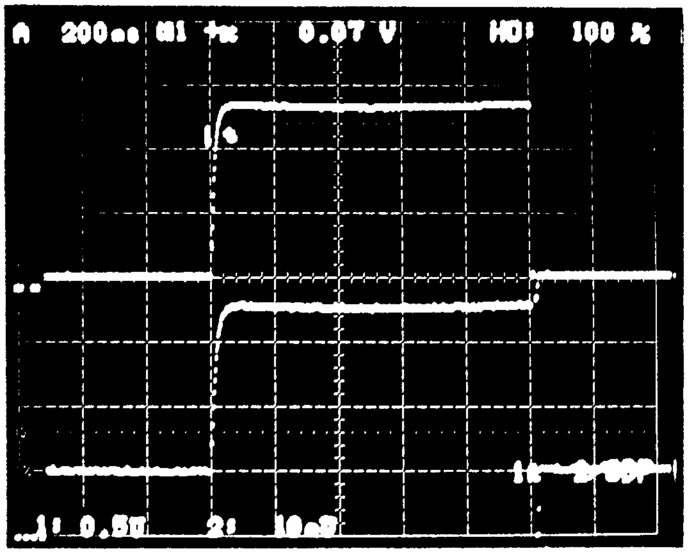

Fig.4 Typical current wave form shared to the resistive shunt. Top trace is the voltage of the shunt resistor and bottom trace is the output of an isolation amplifier to monitor current.

The FET shunt uses the metal oxide semiconductor field-effect transistor (MOSFET) parallel to the magnet coil and the current shared to the shunt is specified by the reference signal (as shown in Fig.5) from the digital-to-analog converter (DAC). The allowable maximum shunt current is limitted by the saturation of MOSFET. The 25A current sharing is attained for 4.1 V terminal voltage. Fig.6 shows the typical current wave form shared to the FET shunt. The sharing duration is 60 sec for this current by using the heat radiating fin.

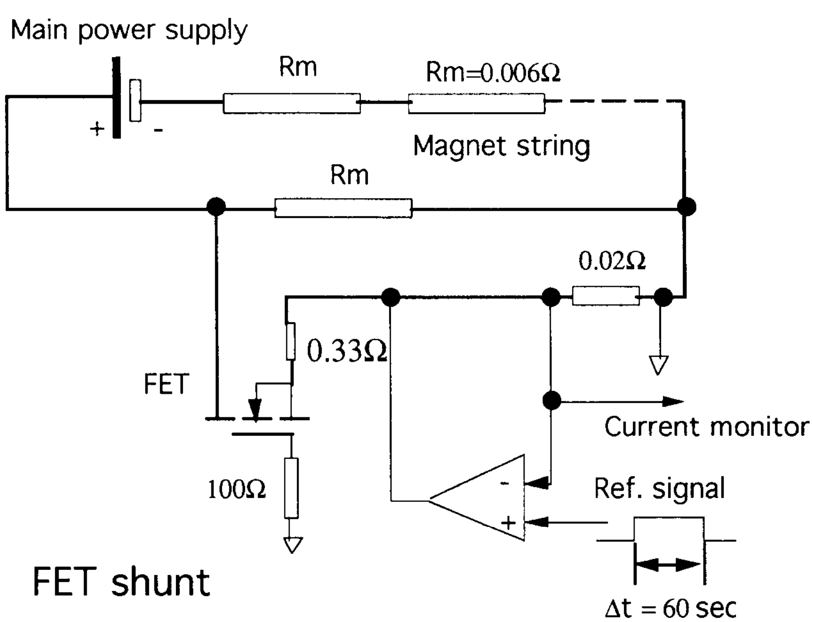

Fig.5 Simplified model circuit of the FET current shunt.

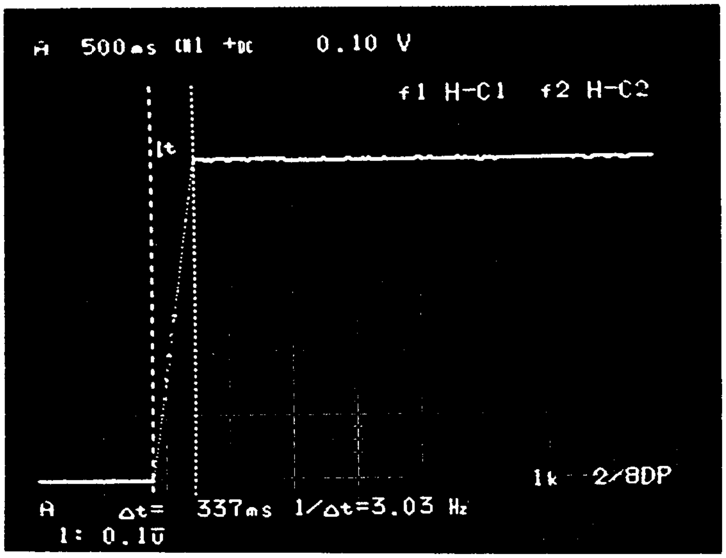

Fig.6 Current wave form shared to the FET shunt. Im=675A and Is=25A. The magnet terminal voltage is 4.1V which is same as the shunt voltage. Rise time of the shunt current is 0.34 sec and the duration of flattop is 60 sec which is enough for several BPM readings.

Either of two types of the current shunt can be incorporated into the beam control system to predict the quadrupole misalignments. Applying the current shunt to every quadrupole sequentially for a short time during the BPM readings can be stored, a complete set of data for BBA are obtained. For this purpose, the individual resistive current shunt will be required to every quadrupole because the shunt resistivity differs by the cable length to the quadrupole magnet. While the FET current shunt dose not depend on the cable length, so the multiplex system can be economically constructed using only one current shunt.

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