Universal longitudinal wake for very short bunches

Gennady Stupakov

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timer

Introduction

- In several problems it was observed that the longitudinal wake in a round pipe at very short distances approaches the value $w_0 = 4/b^2$. Examples: resistive wall wake, periodic accelerating cavities, surface with periodic corrugations, surface with a dielectric coating.

  At the same time, the slope of the transverse wake becomes $w_t' = 8/b^4$.
- These results do not depend on the surface properties (wall conductivity, corrugation size, dielectric constant, etc.).
- Why do we have those universal wakes at very short distances? It is not accidental.

  I will present here arguments that lead to a universal boundary condition which gives these very-short-range wakes.

Universal boundary condition, long. wake

For a resistive wall, from the Leontovich boundary conditions it follows that on the wall $E_z \propto \sqrt{\varpi} H_\theta$, and the Poynting vector $S_r = (c/4\pi) E_z H_\theta \propto \sqrt{\varpi} H_\theta^2$. The energy absorbed in the wall

$$\int d\varpi S_r \propto \int d\varpi \sqrt{\varpi} H_\theta^2$$

diverges unless $H_\theta \to 0$ when $\varpi \to \infty$. Hence the boundary condition at high frequencies (short distances) is $H_\theta = 0$.

For periodic accelerating structures one should use the diffraction model for energy losses. This gives losses at the wall, due to diffraction, $P \propto H_\theta^2 / \sqrt{\varpi}$. Again, $H_\theta \to 0$ when $\varpi \to \infty$, to keep the total radiated energy finite.

Longitudinal impedance and wake

Let us find the impedance and the wake in this limit. In ω representation we need to consider a current $Ie^{-i\omega(t-z/c)}$ on axis of the pipe.

$$\frac{1}{r}\frac{\partial}{\partial r}rH_\theta = i\omega E_z$$

We know that for an ultrarelativistic particle E_z does not depend on r.

$$H_\theta = \frac{1}{2c}i\omega r E_z + \frac{2I}{rc}$$

From the boundary condition $H_\theta|_{r=b} = 0$ we have

$$E_z = \frac{4I}{i\omega b^2}$$

which gives the impedance

Longitudinal impedance and wake

$$Z = -\frac{E_z}{I} = \frac{4i}{\omega b^2}$$

This is capacitive impedance.

$$w = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega Z(\omega) e^{-i\omega s/c} = \frac{2i}{\pi b^2}\int \frac{d\omega}{\omega} e^{-i\omega s/c}$$

$$w(s) = \frac{4}{b^2} h(s)$$

Transverse wake and plans

- I also derived the transverse wake result $w_t' = 8/b^4$ assuming the boundary conditions $H_\theta = 0$ and $E_\theta = 0$ on the wall. The latter is equivalent to requiring $H_r = 0$ (because at high frequencies $H_r = E_\theta$).
- I am working on the problem of the transient build-up of the longitudinal wake when the length of the system is comparable to the catch-up distance.