Space Charge Effects on RF Zero-phasing Bunch Profile Measurements

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BIG meeting

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Introduction

- SDL microbunching observations (T. Shaftan's talk yesterday)

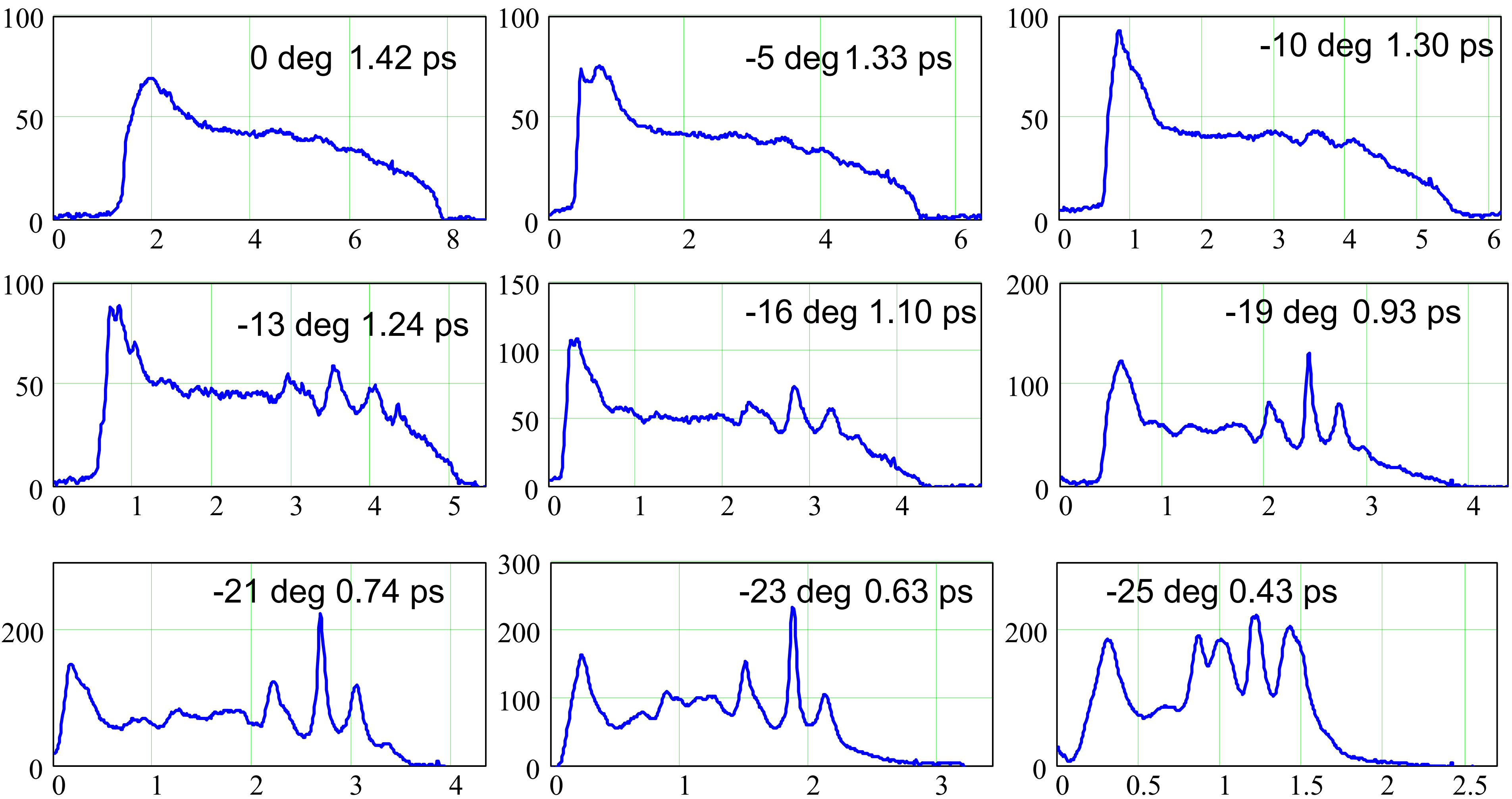
- RF zero-phasing

- Comments on CSR

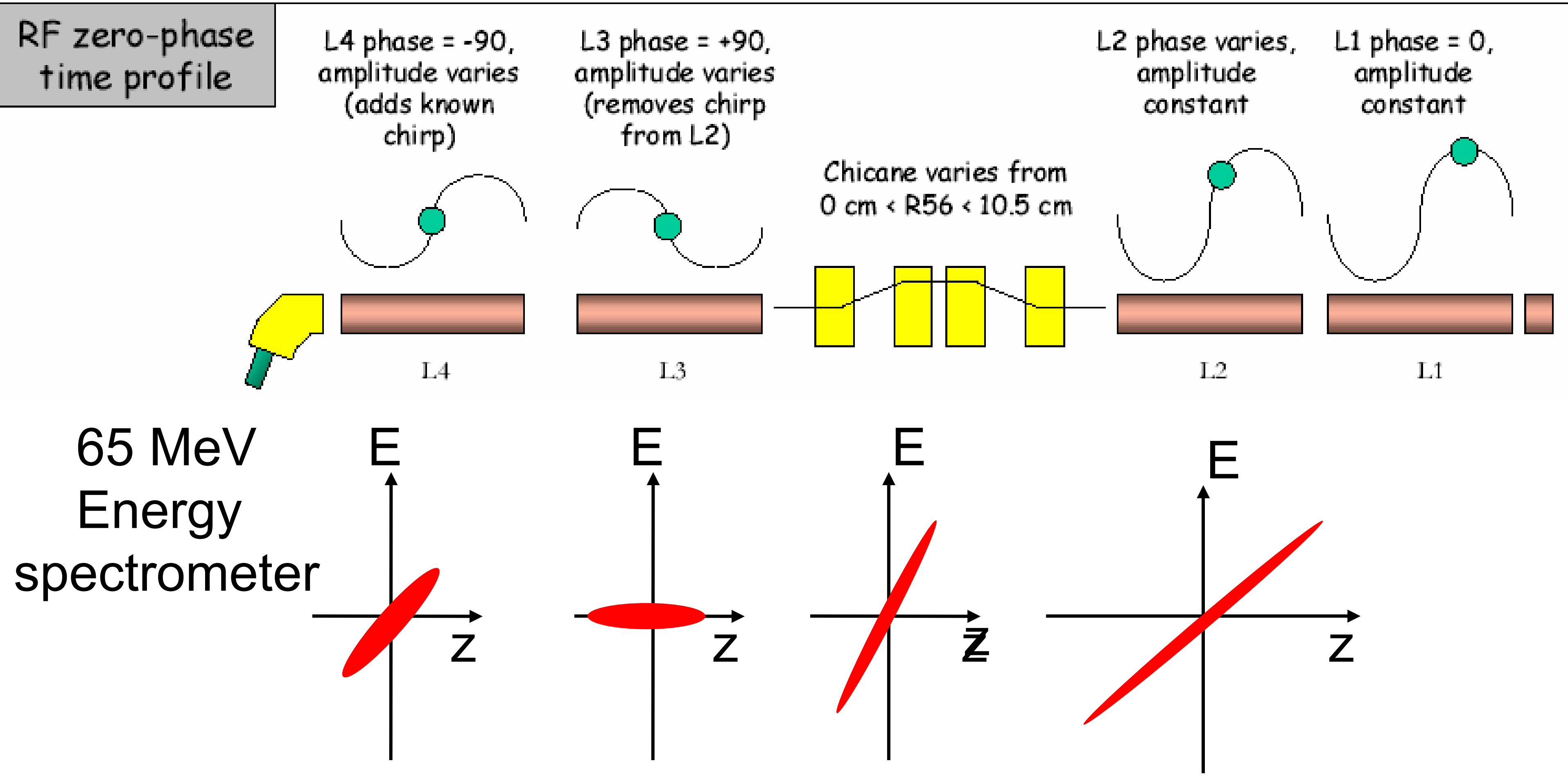
- Space-charge

- Discussion and conclusion

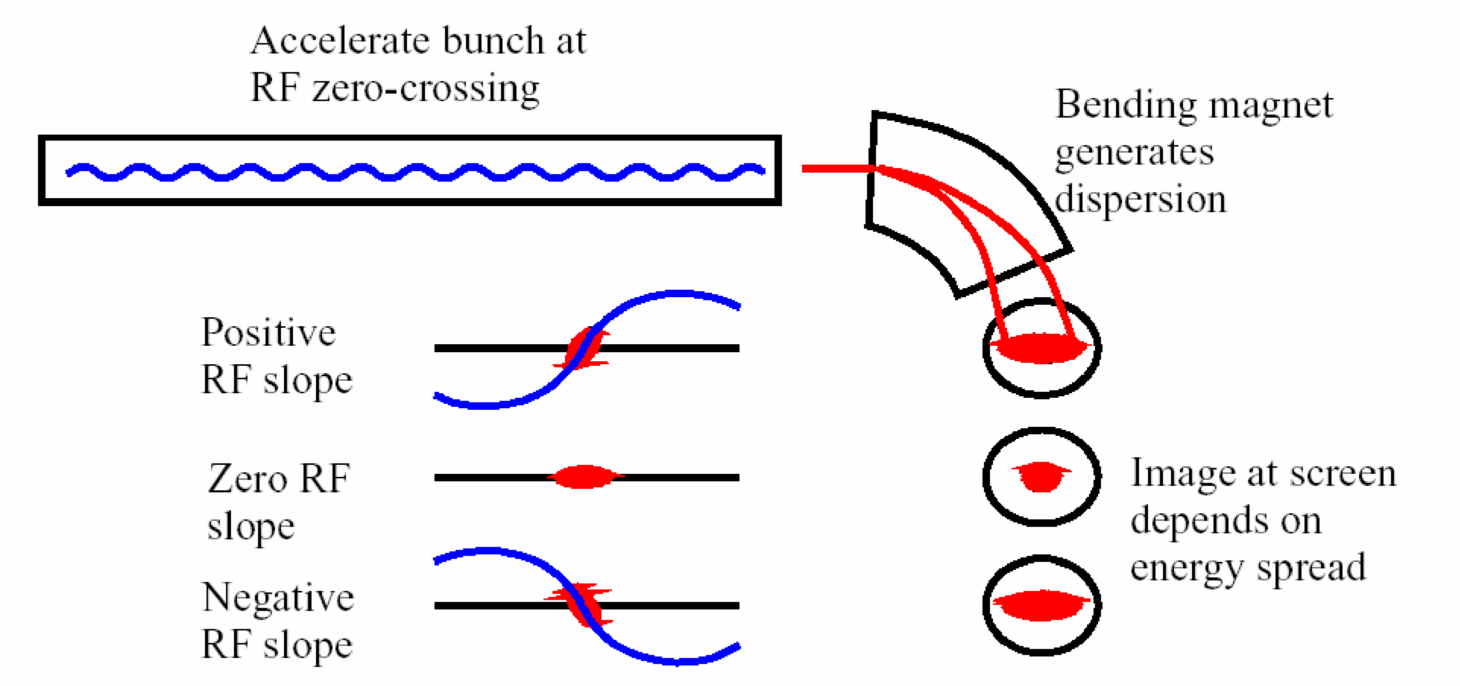
SDL Microbunching Measurements (T. Shaftan et al.)

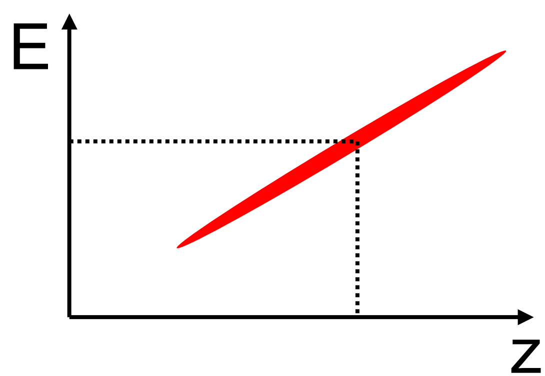


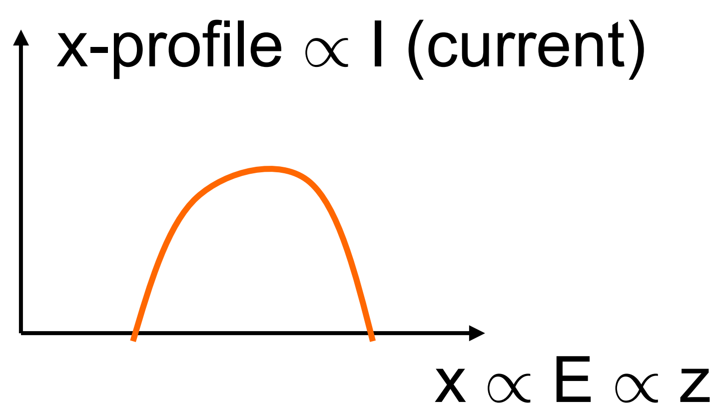
SDL Experimental Setup



RF Zero-phasing

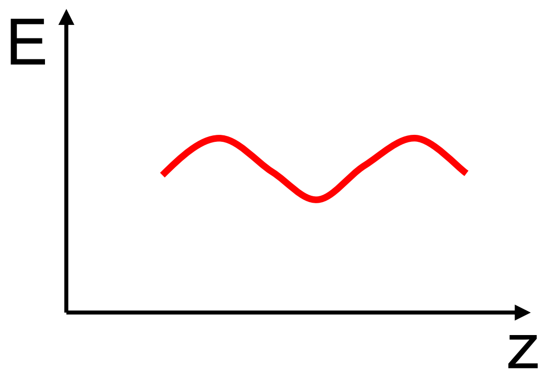


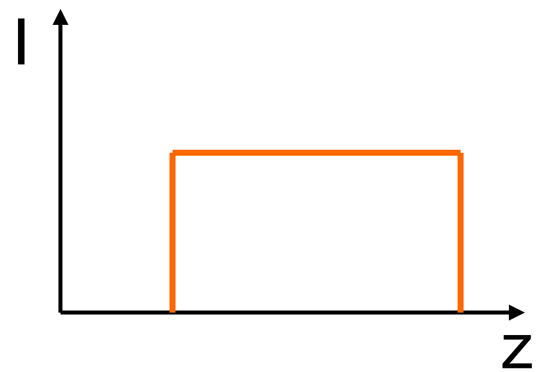




Phase Space Distortion

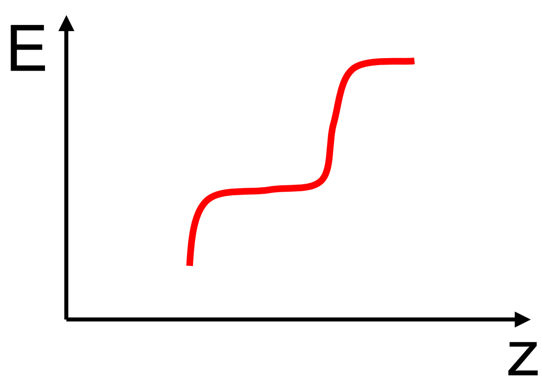
- Distorted longitudinal phase space negates the linear relation between energy and time!

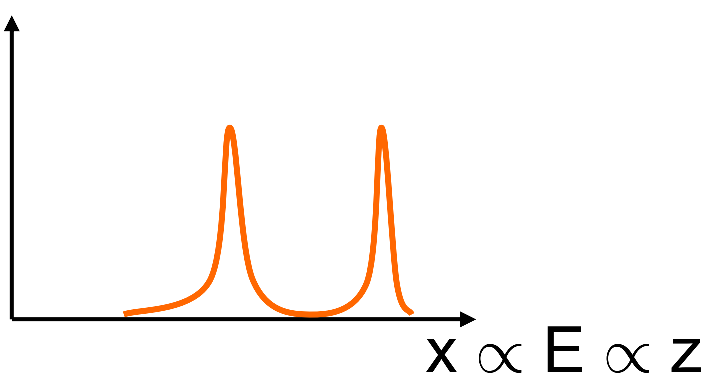




RF zero-phasing

x (or energy) profile





Energy deviation = chirp + sinusoidal modulation

$$\delta = \Delta E/E = hz + \delta_0 \sin(kz),$$

Total charge

$$Q = \int I(z)dz = \int f(\delta)d\delta$$
$$= \int f(\delta)\left[1 + \frac{\delta_0 k}{h}\cos(kz)\right] h dz$$

Energy profile

$$f(\delta) \approx \frac{I(z)}{h}\left[1 - \frac{\delta_0}{h}\cos(kz)\right]$$

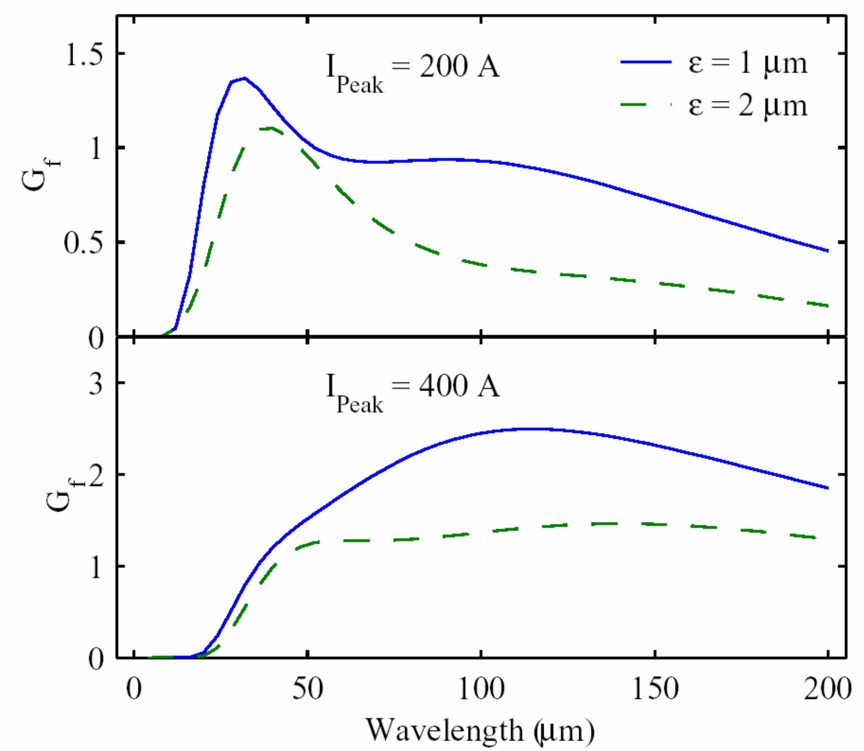
or $$f(\delta) \approx \frac{I(\delta/h)}{h}\left[1 - \frac{\delta_0 k}{h}\cos(k\delta/h)\right]$$

magnification $\delta_0 \frac{E}{E_{rf}\cos\phi}\frac{\lambda_{rf}}{\lambda} \sim 1000\ \delta_0$ for $\lambda = 100\ \mu$m

Beam size and intrinsic energy spread ➔ resolution limit

CSR Microbunching in the SDL Chicane

- One possibility is the amplification of current modulation in the chicane due to CSR (H. Loos, et al., EPAC2002)

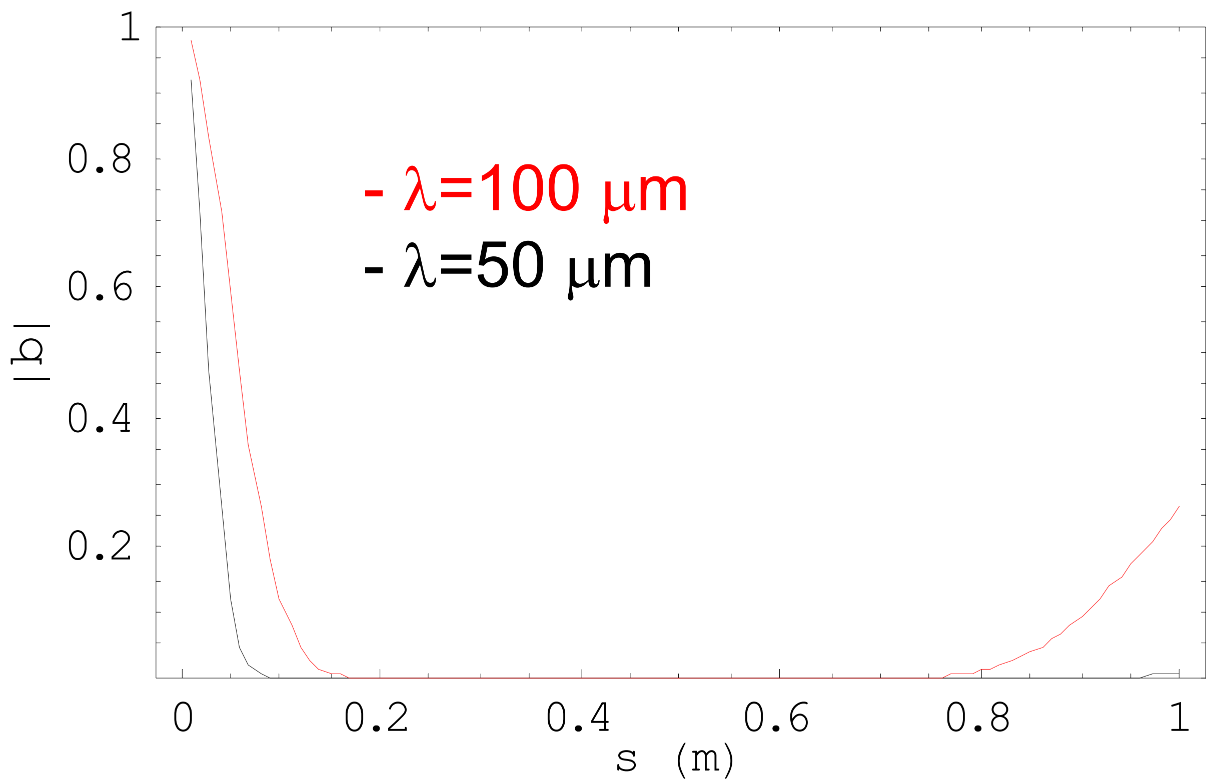


- Very mild gain (~1), if any
- A small but well-defined energy modulation

CSR Microbunching in the Spectr. Dipole

CSR Microbunching in the Spectr. Dipole

• Another possibility is the CSR effect in the spectrometer dipole



• Current modulation (|b| is the modulation amp.) smears out rapidly by emittance

• Impedance effects in bends are limited by an interaction distance $\rho/(k\sigma_x) \sim 0.1$ m

Space Charge Force in the Linac

- Longitudinal space charge field

$$E_s \sim \frac{2}{c\gamma^2}\frac{dI}{dz} \sim \frac{2}{c\gamma^2}\frac{I_0|b|}{\lambda/(2\pi)}$$

- For SDL, I_0=300 A after compression, γ=130 (65 MeV), λ=100 μm, |b|=0.01 (1% modulation) ➔ E_s ~ 0.7 kV/m

- After 10 m drift space, the accumulated energy change due to space charge is 7 keV, or 10^{-4} of total energy

- Energy (horizontal) profile modulation after zero-phasing is magnified 1000 times, or 10% modulation

➔ a gain of 10 if one interpretates the energy (horizontal) profile as the current profile

Space Charge Impedance

Space Charge Impedance

- For a round, parallel electron beams with a uniform transverse cross section of radius r_b, the longitudinal space charge impedance in vacuum

$$Z(k) = \frac{4i}{kr_b^2}\left[1 - \frac{kr_b}{\gamma}K_1\left(\frac{kr_b}{\gamma}\right)\right]R(k)$$

$$\approx \frac{4i}{kr_b^2} \text{ if } \frac{kr_b}{\gamma} \gg 1$$

$$\approx \frac{ik}{\gamma^2}\left(1 + 2\ln\frac{1.1\gamma}{r_b k}\right)R(k) \text{ if } \frac{kr_b}{\gamma} \ll 1$$

- R(k) is a reduction factor due to the transverse dependence of the longitudinal field (close to unity)
- Free space approximation is good when

$\gamma\, \lambda/(2\pi) \sim 2$ mm $\ll$ beam pipe radius

Space Charge Oscillation

- Equation of motion (no acceleration, γ constant)

$$\frac{dz}{ds} = \frac{1}{\gamma^2}\delta,$$
$$\frac{d\delta}{ds} = -\frac{I_0}{\gamma I_A}|b(k)Z(k)|\cos(kz+\psi)$$

- This looks like synchrotron oscillation
- The self-consistent solution is the space charge oscillation

frequency $\Omega = c\left[\frac{I_0}{\gamma^3 I_A}k|Z(k)|\right]^{1/2}$

= plasma frequency in 1D limit

- Landau damping is negligible in the straight section: slippage ~ $\Delta L\ \sigma_\delta/\gamma^2 << 1\ \mu$m for a drift length ΔL~10 m

Gain in Horizontal Modulation

- Assume a small current modulation $b_0(k)$ after bunch compressor, but no initial energy modulation

- Evolution of current and energy modulation in the linac

$$b(k;s) = b_0(k)\cos(\Omega s/c),$$

$$\delta(k;s) = -\frac{I_0}{\gamma I_A} Z(k) b_0(k) \frac{\sin(\Omega s/c)}{\Omega/c}$$

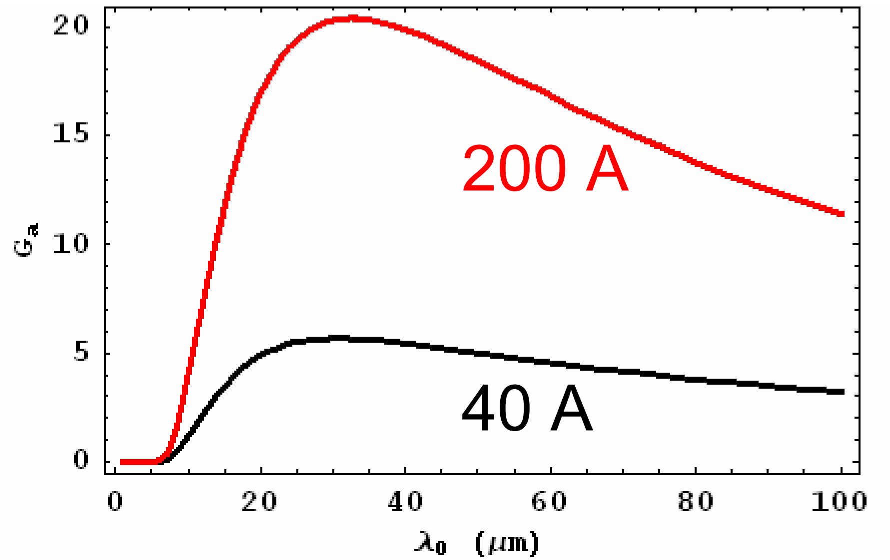
- The effective interaction distance is the oscillation period ~ 10 m, two orders of magnitude higher than that in bends!

- RF zero-phasing “sees” both current modulation and magnified energy modulation, and hence more modulation than just current modulation (➔“gain”)

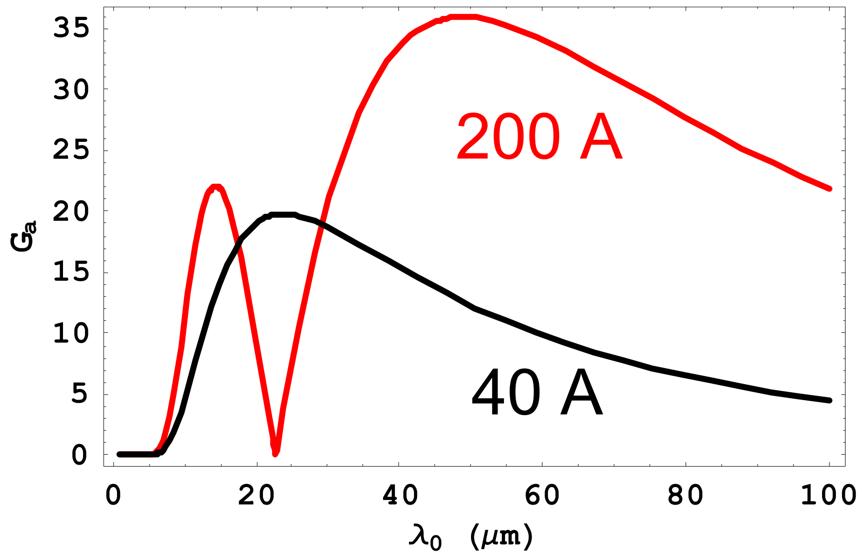
- Define “gain” = x modulation amplitude/current modulation

$$G_a = \left| \cos(\Omega\Delta L/c) - \frac{k}{h}\left[\frac{\gamma I_0}{I_A}\frac{|Z(k)|}{k}\right]^{1/2} \sin(\Omega\Delta L/c) \right|$$

$$\times \exp\left(-\frac{k^2\sigma_\delta^2}{2h^2} - \frac{k^2\sigma_x^2}{2h^2\eta^2}\right)$$



large linac beam (1 mm)



small linac beam (250 μ m)

Discussion

• Theory calculates “gain”, experiments observe modulation amplitude (connected by the initial condition)

• BC increases the peak current ➔ “gain” spectrum RF curvature generates spikes after compression, hence can increase the initial current modulation

• The origin of modulation may be traced back to the drive laser (ripples), and space charge oscillation can occur at lower energy prior to BC (+Landau damping)

• Present analysis of space charge oscillation is valid under adiabatic acceleraton ($d\gamma/(ds\ \gamma_0) << \Omega/c$)

Summary

• RF zero-phasing bunch profile measurement (energy spectrometry of a chirped beam) is very sensitive to the longitudinal phase space distortion

• CSR not critical in SDL microbunching observations

• Space charge force can set up a long. phase space distortion, leading to horizontal modulations much stronger than current modulations it attemps to measure

• Bunch compression can enhance the initial current spikes and any collective effect downstream

Next Steps

- Modeling
  - space charge dynamics at lower energy
  - Parmela simulation of laser ripple effects for different charges (C. Limborg)
  - implement space charge impedance/wake in elegant/numerical tracking (T. Shaftan)

  ➔sources of modulations and their magnitudes

- Experiments
  - dependence on beam size
  - measurements at higher energy, different rf phase
  - modulates laser profile

- Other effects and possibilities