Resistive wall wake with ac conductivity and the anomalous skin effect

Karl Bane and Gennady Stupakov

August 6, 2004

Introduction

•resistive wall wake is a limiting effect in the LCLS undulator, with the induced $\Delta E \sim \rho$, the Pierce parameter

•can add effects of ac conductivity (see K. Bane and M. Sands, SLAC-PUB-95-7074) and anomalous skin effect (Reuter and Sondheimer, Proc Royal Soc., 1948) to resistive wall model

Ac conductivity

•Drude free-electron model of conductivity (1900): conduction electrons are treated as an ideal gas, whose velocity distribution was given in equilibrium at temperature *T* by the Maxwell-Boltzmann distribution

•Sommerfeld (1920's) replaced the distribution by the Fermi-Dirac distribution

•this free-electron model correctly describes many electrical and thermal properties of metals

Parameters

- density of conduction electrons n (~10^{22}/cm^3)
- collision time (or mean free time, or relaxation time) τ (~10^{-14} s)
- dc conductivity $\sigma = ne^2\tau/m$
- ac conductivity $\tilde{\sigma} = \dfrac{\sigma}{1 - i\omega\tau}$
- Fermi velocity v_F (~$0.01c$)
- mean free path $\ell = v_F\tau$
- note that σ/τ, ℓ/τ nearly independent of temperature

10 Chapter 1 The Drude Theory of Metals

Table 1.3
DRUDE RELAXATION TIMES IN UNITS OF 10^{-14} SECOND[a]

ELEMENT	77 K	273 K	373 K
Li	7.3	0.88	0.61
Na	17	3.2	
K	18	4.1	
Rb	14	2.8	
Cs	8.6	2.1	
Cu	21	2.7	1.9
Ag	20	4.0	2.8
Au	12	3.0	2.1
Be		0.51	0.27
Mg	6.7	1.1	0.74
Ca		2.2	1.5
Sr	1.4	0.44	
Ba	0.66	0.19	
Nb	2.1	0.42	0.33
Fe	3.2	0.24	0.14
Zn	2.4	0.49	0.34
Cd	2.4	0.56	
Hg	0.71		
Al	6.5	0.80	0.55
Ga	0.84	0.17	
In	1.7	0.38	0.25
Tl	0.91	0.22	0.15
Sn	1.1	0.23	0.15
Pb	0.57	0.14	0.099
Bi	0.072	0.023	0.016
Sb	0.27	0.055	0.036

[a] Relaxation times are calculated from the data in Tables 1.1 and 1.2, and Eq. (1.8). The slight temperature dependence of n is ignored.

(Ashcroft and Mermin, p. 10)

Relaxation times τ in [10^{-14} s]

How good is the free electron model for real metals?

Im(ε) for Cu

Im(ε) for Ag

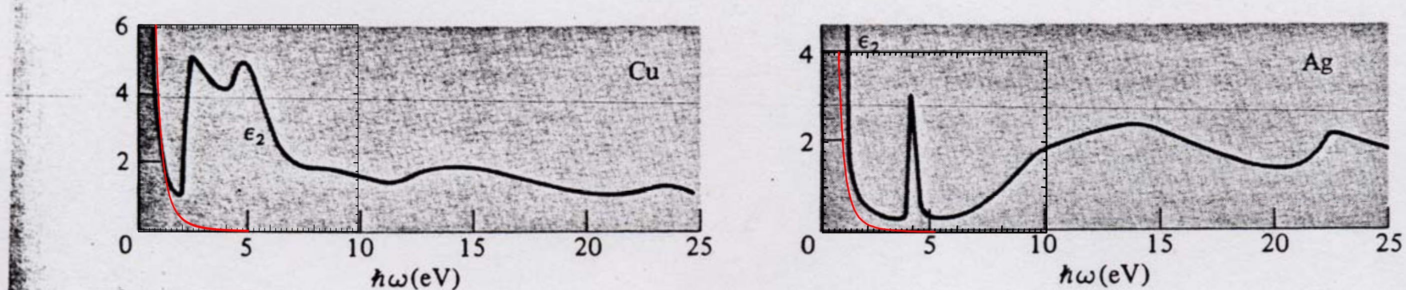


Figure 15.12
The imaginary part of the dielectric constant, $\epsilon_2(\omega) = \text{Im}\,\epsilon(\omega)$ vs. $\hbar\omega$, as deduced from reflectivity

Im(ε) from reflectivity measurements (Ashcroft/Mermin, p. 297)

- note: $\epsilon(\omega) = 1 + \frac{4\pi i\tilde{\sigma}}{\omega}$ so $Im(\epsilon) = \frac{4\pi\sigma}{\omega}\frac{1}{(1+\omega^2\tau^2)}$
- k= 1/0.1μm⇔$\hbar\omega$=2eV, red light; (also a/γ= 1/0.1μm)

Calculation of wake--dc conductivity

- impedance (see A. Chao): $$Z = \left(\frac{Z_0}{2\pi a}\right) \frac{1}{\frac{\lambda}{k} - \frac{ika}{2}}$$

with $\lambda = \sqrt{\frac{2\pi\sigma|k|}{c}}\,[i + sgn(k)]$

- inverse Fourier transform to find wake

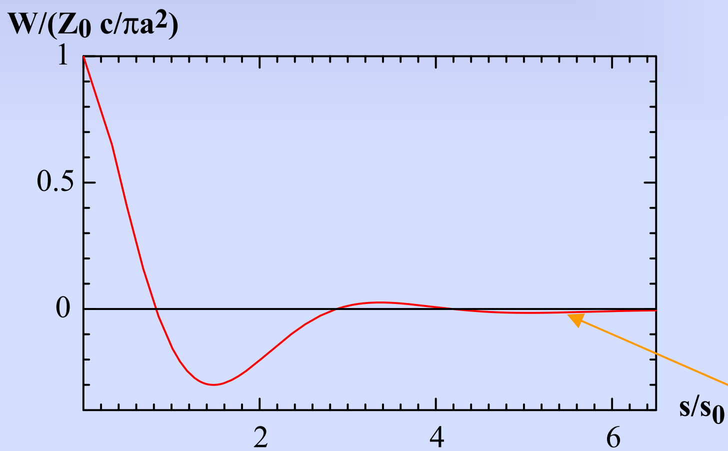
- general solution is composed of a resonator term and a diffusion term

•General solution

$$W = \frac{4Z_0 c}{\pi a^2}\left(\frac{e^{-s/s_0}}{3}\cos\frac{\sqrt{3}s}{s_0} - \frac{\sqrt{2}}{\pi}\int_0^\infty \frac{dx\, x^2 e^{-x^2 s/s_0}}{x^6+8}\right)$$

$$s_0 = \left(\frac{2a^2}{Z_0\sigma}\right)^{\frac{1}{3}}$$

(for Cu with a= 2.5mm, s_0= 8.1μm)



wake for dc conductivity

long range wake:

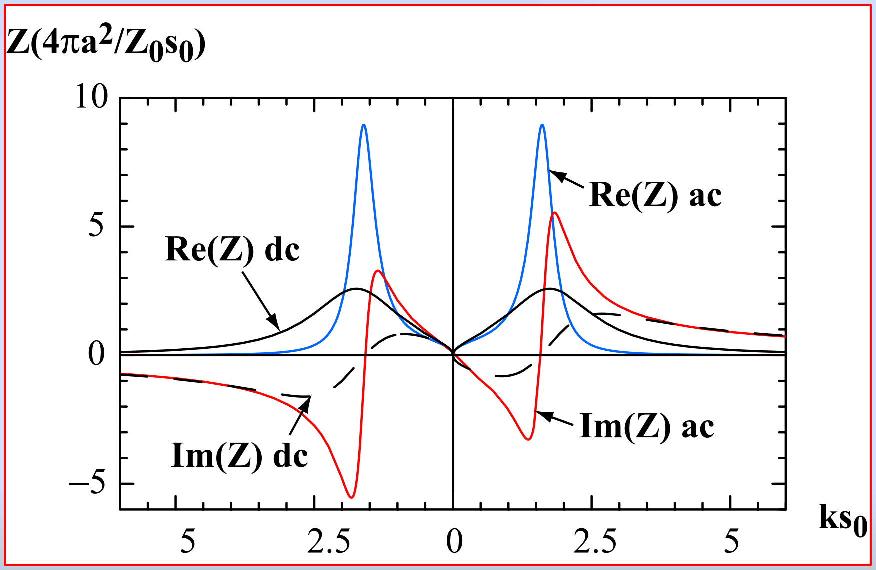
$$W(s) = -\frac{c}{4\pi^{3/2}a}\sqrt{\frac{Z_0}{\sigma}}\frac{1}{s^{3/2}}\ ,$$

Ac conductivity

- new parameter $\Gamma = c\tau/s_0$.

- for Cu with beam pipe radius a= 2.5 mm, s_0= 8 μm, $c\tau$= 8 μm, Γ= 1.0; for Al, s_0= 9.3 μm, $c\tau$= 2.4 μm, Γ= 0.26.

- for ac conductivity replace σ with $\tilde{\sigma}$ in impedance; then again take inverse Fourier transform for wake



impedance

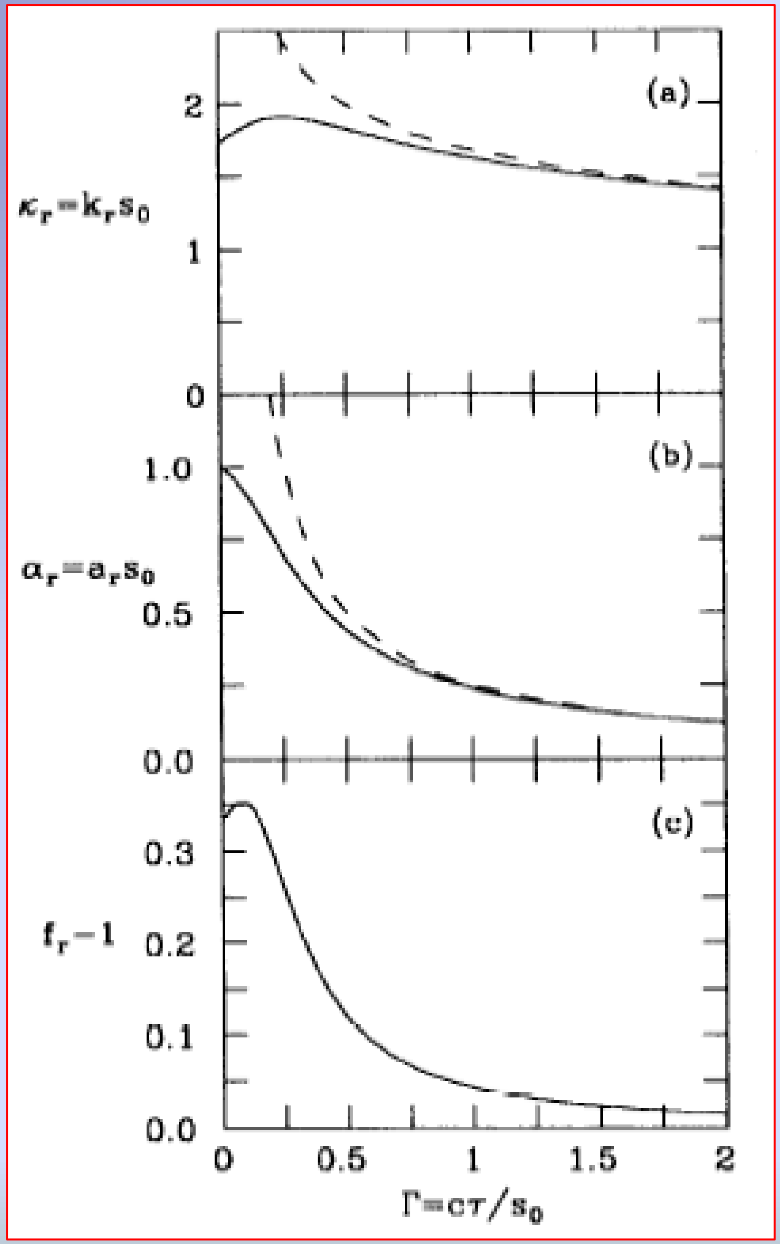
note: Re(Z)~0 for k≥ 1/2μm

•wake $W_z(z)$ is composed of a resonator and a diffusion component

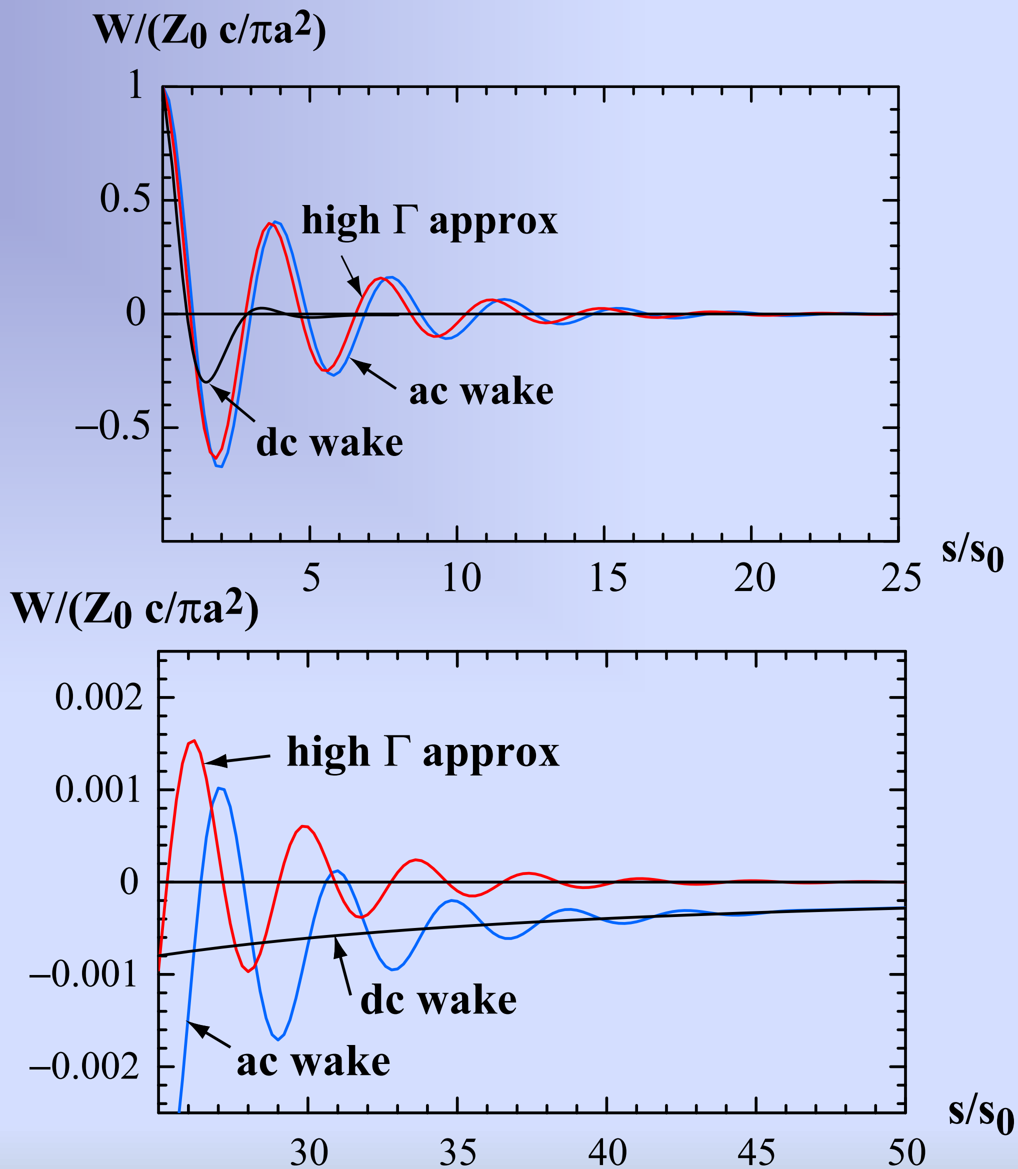
•for $\Gamma \gtrsim 1$, can approximate

$$W_z(s) = \frac{Z_0 c}{\pi a^2} e^{-s/4c\tau} \cos\left[\sqrt{\frac{2\omega_p}{ac}} s\right]$$

with the plasma frequency $\omega_p = \sqrt{4\pi\sigma/\tau}$

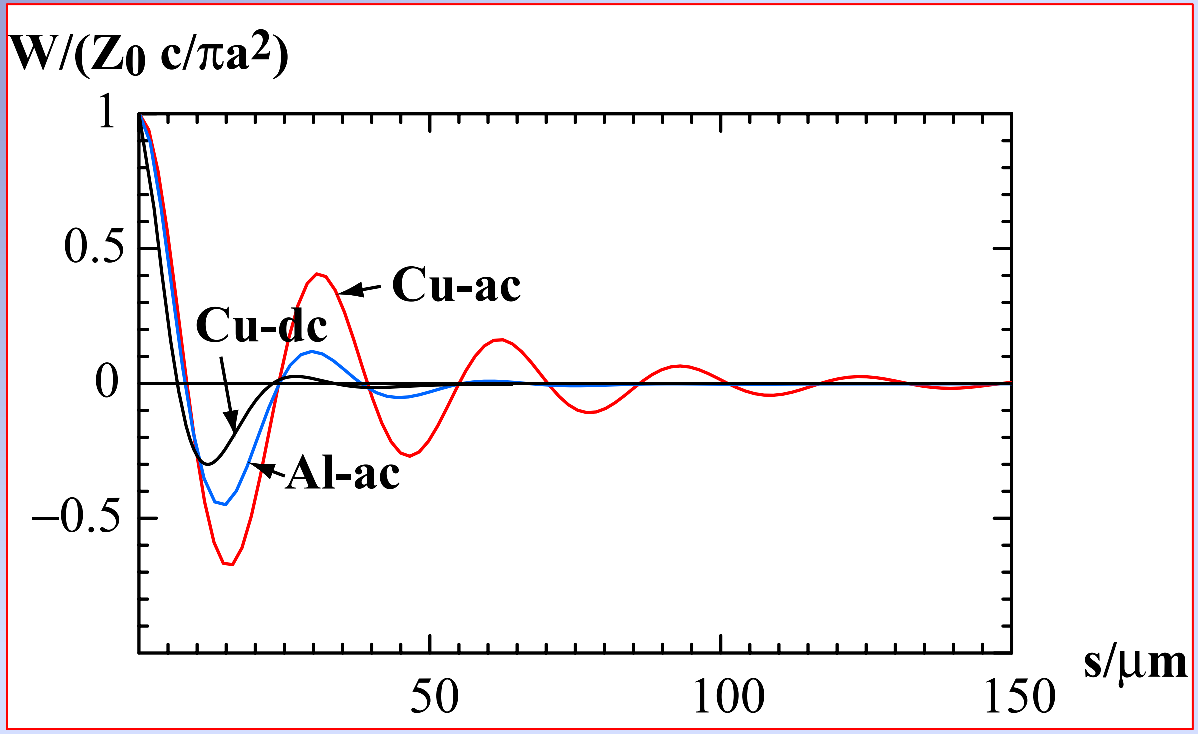


for the resonator component of the wake: the frequency κ_r, damping factor α_r, and amplitude f_r; the large Γ analytical formula is given by dashes (from K. Bane and M. Sands).

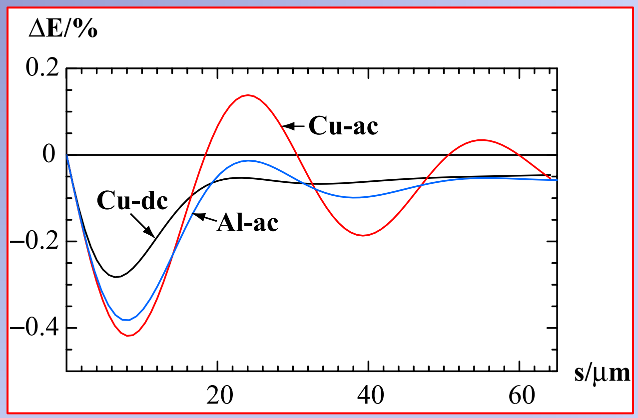


s_0= 8 μm

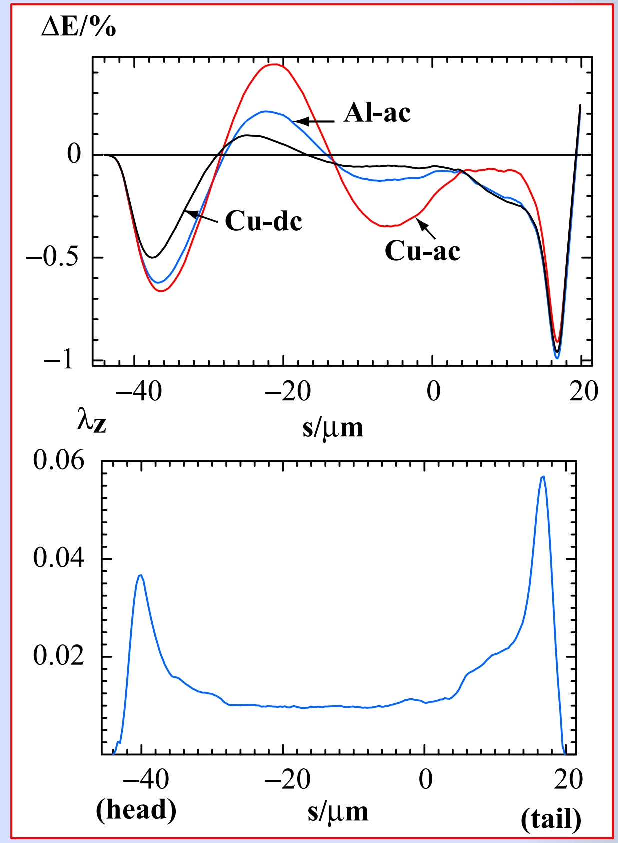
ac wake with high Γ approximation



Point charge wake



Induced energy change for rectangular bunch with full length of 65 μm



charge—1 nC, energy—14 GeV, tube radius—2.5 mm, tube length—130 m

Induced energy change (top) for LCLS bunch shape (bottom).

Anomalous skin effect (Reuter and Sondheimer)

when $\ell > \delta = c/\sqrt{2\pi\omega\sigma}$ the skin depth, the anomalous skin effect occurs, the fields don't drop exponentially with distance into metal

in principle this can happen at low temperatures or high frequencies; nevertheless, "It is evident that no appreciable departure from the classical behaviour is to be expected at ordinary temperatures, so that the anomalous skin effect is essentially a low-temperature phenomenon"—Reuter and Sondheimer.

for Cu at room temperature, ℓ = 0.04μm and for k = 1/20μm, δ = 0.04μm

sketch of solution method

•consider semi-infinite metal, with surface in *xy* plane, and positive *z* into metal

•the distribution function of electrons is written in form: $f = f_0 + f_1(\mathbf{v}, z)$, with f_0 the Fermi-Dirac distribution

•combining Maxwell's equation and Boltzmann's equation, obtain

$$\frac{\partial f_1}{\partial z} + \frac{1 + i\omega\tau}{\tau v_z} f_1 = \frac{e}{m v_z} \frac{\partial f_0}{\partial v_x} E(z)$$

•without first term get classical solution

•also classical if $c_1 \tau \ll \omega\tau[1 + 1/(\omega^2\tau^2)]^{3/4}$; at high frequencies is classical if path travelled by electron during one period of the field is small compared to the penetration depth $\delta\sqrt{\omega\tau}$ [c_1 is a constant]

Anomalous skin effect

results given in terms of the surface impedance $Z = R + i\,X$

$$\frac{R}{R_{cl}} = \frac{4}{\pi}\sqrt{\frac{2\alpha}{3}}\frac{\omega\tau \mathrm{Re}(I) - \mathrm{Im}(I)}{(1+\omega^2\tau^2)\sqrt{-\omega\tau + \sqrt{1+\omega^2\tau^2}}}$$

$$\frac{X}{X_{cl}} = \frac{4}{\pi}\sqrt{\frac{2\alpha}{3}}\frac{\mathrm{Re}(I) + \omega\tau \mathrm{Im}(I)}{(1+\omega^2\tau^2)\sqrt{\omega\tau + \sqrt{1+\omega^2\tau^2}}}$$

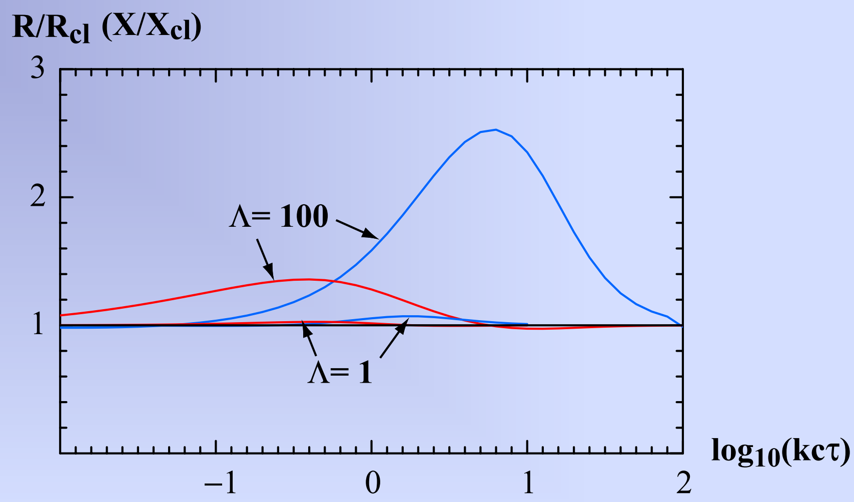
$$I = \int_0^\infty \frac{dt}{t^2 + \frac{i\alpha}{(1+i\omega\tau)^3}\kappa(t)}$$

$$\kappa(t) = 2[(1+t^2)\tan^{-1} t - t]/t^3$$

$$R_{cl} = \sqrt{\frac{2\pi\omega}{c^2\sigma}}\sqrt{-\omega\tau + \sqrt{1+\omega^2\tau^2}}$$

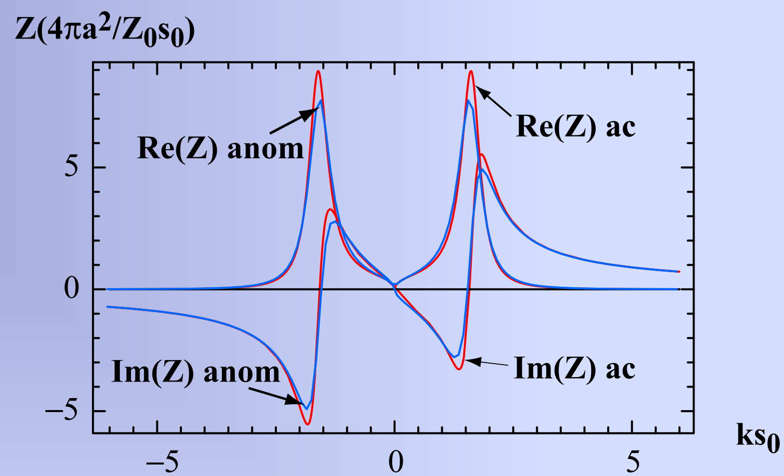
$$X_{cl} = \sqrt{\frac{2\pi\omega}{c^2\sigma}}\sqrt{\omega\tau + \sqrt{1+\omega^2\tau^2}}$$

$\alpha=1.5\ell^2/\delta^2$; normalized parameter $\Lambda= \alpha/kc\tau$; for Cu at room temperature $\Lambda= 3.4$



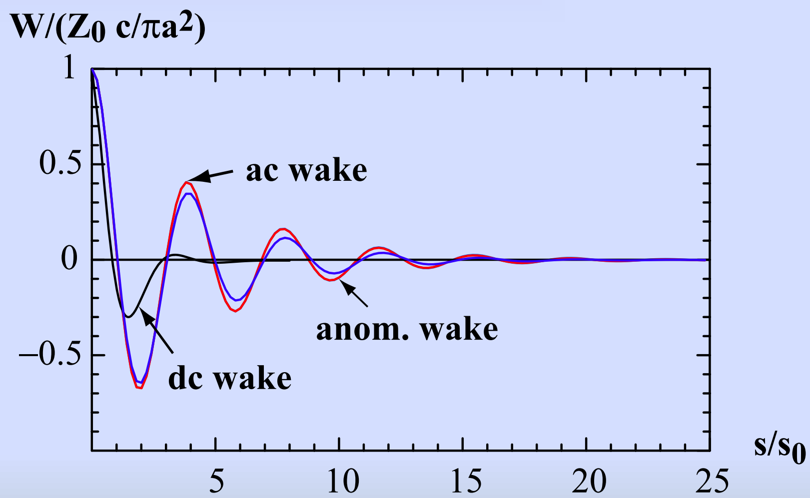
sensitivity of anomalous skin effect to frequency; given are R/R_{cl} (blue) and X/X_{cl} (red).

- note: for $\Lambda= 3.4$, peak of $R/R_{cl}= 1.2$
- to find impedance, set $\lambda = \dfrac{4\pi|k|}{R\,\mathrm{sgn}(k) - iX|k|}$; for wake again take inverse Fourier transform



$\Lambda = 3.4$

impedance for a=2.5mm Cu tube
including anomalous skin effect



wake