Implicit scheme for wake field calculations

A.Novokhatski

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Beam Instability Group Meeting,

SLAC, Stanford

VLEPP Linear Collider

A.Novokhatski "Implicit Scheme for Wake Field Calculations"

All Union Conference On Particle Accelerators

ГОСУДАРСТВЕННЫЙ КОМИТЕТ
ПО ИСПОЛЬЗОВАНИЮ АТОМНОЙ ЭНЕРГИИ СССР
ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

ТРУДЫ
ШЕСТОГО ВСЕСОЮЗНОГО
СОВЕЩАНИЯ
ПО УСКОРИТЕЛЯМ
ЗАРЯЖЕННЫХ ЧАСТИЦ

Том I

Dubna
1978

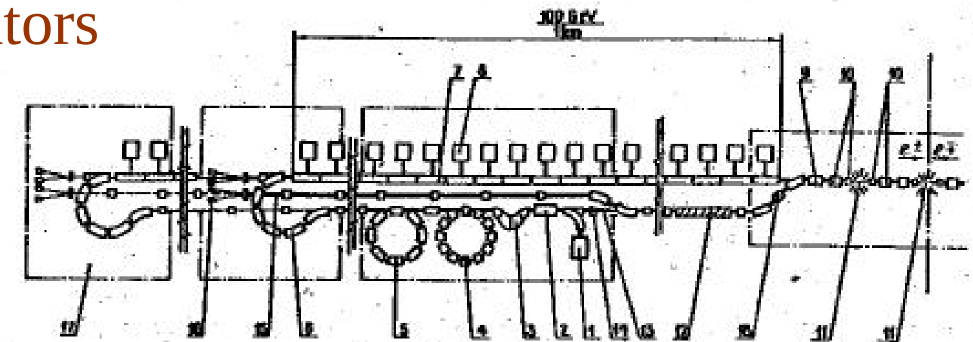

Рис.1

1 – Начальный инжектор. 2 – Промежуточный ускоритель. 3 – Дебанчер. 4 – Накопительное кольцо. 5 – Охладитель-инжектор. 6 – Банчер. 7 – Ускоряющие секции. 8 – Источник СВЧ. 9 – Импульсный дефлектор. 10 – Фокусирующие линзы. 11 – Места встречи. 12 – Спиральный ондулятор. 13 – Пучок γ-квантов. 14 – Конверсионная мишень. 15 – Остаточный электронный пучок. 16 – Эксперименты с электронными (позитронными) пучками со стационарной мишенью. 17 – Вторая очередь. 18 – Спектрометр.

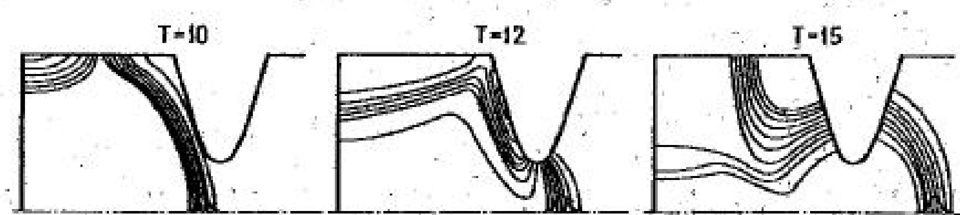

Рис.2. Временная картинка силовых линий электрического поля заряженного сгустка, пролетающего мимо диафрагмы.

Later Presentations

1998

TECHNISCHE UNIVERSITÄT DARMSTADT

KWT -Seminar 1998

Fachgebiet Theorie Elektromagnetischer Felder
Institut für Hochfrequenztechnik
Schloßgartenstraße 8
64289 Darmstadt

Vorträge Kleinwalsertal 30.08-05.09.1998
"Advance in Electromagnetic Research"

International Computational Accelerator Physics Conference

ICAP '98

September 14-18, 1998
Monterey Conference Center
Monterey, California, USA

d NERSC with support from the U.S. DOE

Early Registration Past
Late Poster Abstracts Accepted
Earliest date possible *(Important!)*
.Due November 15, 1998

Longitudinal fields

1 Equations for simulations

Convenient equation for the wake field computer calculations can be derived from the Maxwell equations, which in the structures with cylinder symmetry are

$$\frac{\partial E_z}{c\partial t} = \frac{1}{r}\frac{\partial (rH_\varphi)}{\partial r} - \frac{4\pi}{c} j_z^m \qquad (1)$$

$$\frac{1}{r}\frac{\partial (rE_r)}{\partial r} + \frac{\partial E_z}{\partial z} = 4\pi \varrho^m \qquad (2)$$

$$\frac{\partial H_\varphi}{c\partial t} = -\frac{\partial E_r}{\partial z} + \frac{\partial E_z}{\partial r} \qquad (3)$$

and boundary condition is

$$\vec{n} \times \vec{E} = 0$$

Relativistic bunch

For the wake potential calculations we usually assume, that the beam is moving with the constant velocity. Therefore the charge density ϱ and current density $\vec{j}$ can be described in the way

$$\varrho = \varrho(s,r)\delta(z - ct + s)$$

$$\vec{j} = j_z = \varrho(s,r)V_z\delta(z - Vt + s)$$

where $\varrho(s,r)$ is the charge density inside the bunch.

From the above equations, by integration in radial direction we have

$$\frac{\partial^2}{c^2\partial t^2}\int_0^r E_z r dr = \frac{\partial^2}{\partial z^2}\int_0^r E_z r dr + r\frac{\partial}{\partial r}E_z - 4\pi\left(1 - \frac{V_z^2}{c^2}\right)\int_0^r \varrho(s,r) r dr \quad (4)$$

In the relativistic case, when the velocity of the particles is equal to the speed of light ($V_z = c$), the last term disappears.

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Main Equation

So, we can use the flux Φ of electric field

$$\Phi(t,r,z) = \int_0^r E_z(t,r',z)r'dr'$$

that describes the electromagnetic field components

$$E_z = \frac{1}{r}\frac{\partial}{\partial r}\Phi$$

$$E_r = \frac{1}{r}\frac{\partial}{\partial z}\Phi + \frac{2}{rc}I_z$$

$$H_\varphi = \frac{1}{r}\frac{\partial}{c\partial t}\Phi + \frac{2}{rc}I_z$$

Where I_z is the current of the traveling bunch is

$$\vec{I} = I_z = cq(s)\delta(z-ct+s)$$

$$q(s) = 4\pi\int_0^r \varrho(s,r)r\,dr$$

This flux Φ satisfies the second order equation

$$\frac{\partial^2\Phi}{c^2\partial t^2} - \frac{\partial^2\Phi}{\partial z^2} = r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\Phi}{\partial r}\right)$$

with boundary conditions

$$\vec{n}\bullet\left(\overrightarrow{grad}\Phi - \frac{2}{rc}\vec{I}\right) = 0$$

Explicit Scheme

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2 Explicit Scheme

Usually the explicit scheme of the finite-difference approximation is used

$$\Phi_k^{n+1} - 2\Phi_k^n + \Phi_k^{n-1} = \left(\frac{c\Delta t}{\Delta z}\right)^2 (\Phi_{k+1}^n - 2\Phi_k^n + \Phi_{k-1}^n) +$$
$$+ \left(\frac{c\Delta t}{\Delta r}\right)^2 r\Delta\frac{1}{r}\Delta\Phi_k^n$$

where index n is for time and index k is for z coordinate.

Scheme Stability

The stability conditions we can get from Furrier analyses

$$\Phi \sim e^{i\omega t+i\beta z+i\alpha r}$$

When $r \gg \Delta r$ dispersion relation is

$$\sin^2\frac{\omega\Delta t}{2} = \left(\frac{c\Delta t}{\Delta z}\right)^2 \sin^2\frac{\beta\Delta z}{2} + \left(\frac{c\Delta t}{\Delta r}\right)^2 \sin^2\frac{\alpha\Delta r}{2}$$

And the stability condition is

$$\left(\frac{c\Delta t}{\Delta z}\right)^2 + \left(\frac{c\Delta t}{\Delta r}\right)^2 \leq 1$$

Phase Velocity in Free Finite-difference Domain

If we look for the wave, traveling in z direction, we can notice that the phase velocity in free finite time domain depends upon frequency,

$$V_{ph} = \frac{\omega}{\beta} = c\frac{2}{\beta c \Delta t} arcsin \frac{c\Delta t}{\Delta z} \sin \frac{\beta \Delta z}{2}$$

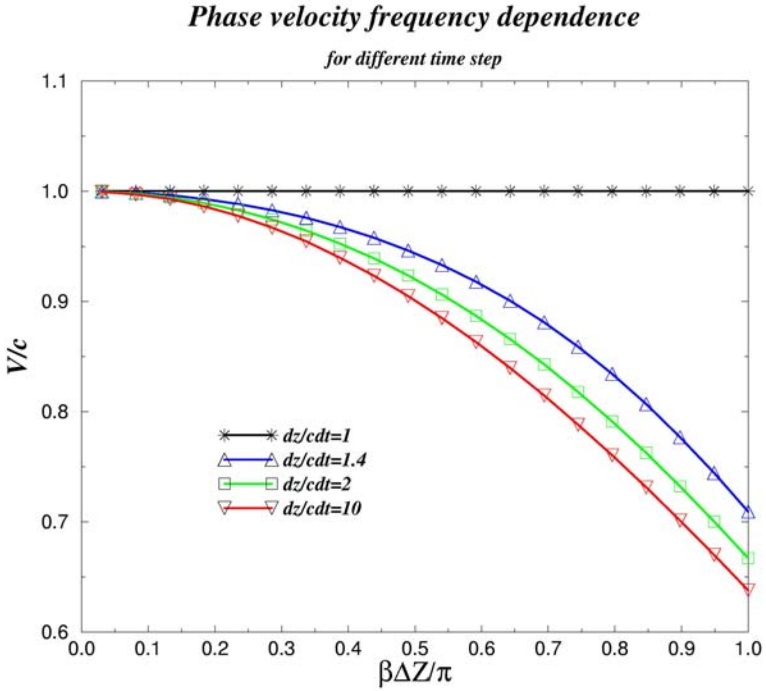
decreasing strongly in the region of maximum value of the wave vector β

$$\beta_{max} = \frac{\pi}{\Delta z}$$

where wavelength λ reaches its minimum value

$$\lambda_{min} = 2\Delta z$$

Dispersion



Phase velocity frequency dependence is given for different ratio of the coordinate step to the time step.

It means, that the finite difference domain has dispersion, that destroys the shape of field, which contains high frequency waves. Usually, the effect of dispersion is in additional high frequency oscillations and diffusion.

Implicit Scheme

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3 Implicit Scheme

Only equivalent step in time and coordinate gives right approximation in high frequency region, but stability condition does not allow to have such step in explicit scheme. So, the solution is in using implicit algorithm.

$$\Phi_k^{n+1} - 2\Phi_k^n + \Phi_k^{n-1} = \left(\frac{c\Delta t}{\Delta z}\right)^2 (\Phi_{k+1}^n - 2\Phi_k^n + \Phi_{k-1}^n) +$$

$$+\frac{1}{2}\left(\frac{c\Delta t}{\Delta r}\right)^2 r\Delta\frac{1}{r}\Delta(\Phi_k^{n+1} + \Phi_k^{n-1})$$

Implicit scheme gives not only stable solution,

but also better "numerical dispersion curve" in the region of minimum critical wavelength, as we can have time step equal to coordinate step $c\Delta t = \Delta z$. This is very important for the calculations of the field of the very short bunches.

$$\cos\omega\Delta t = \frac{\cos\beta\Delta z}{1 + 2\left(\frac{c\Delta t}{\Delta r}\right)^2 \sin^2\frac{\alpha\Delta r}{2}}$$

Solution

Finally, the implicit scheme for the $c\Delta t = \Delta z$ takes the form

$$\Phi_k^{n+1} - \frac{1}{2}(\frac{c\Delta t}{\Delta r})^2 r\Delta\frac{1}{r}\Delta\Phi_k^{n+1} = \Phi_{k+1}^n + \Phi_{k-1}^n - \Phi_k^{n-1} + \\ +\frac{1}{2}(\frac{c\Delta t}{\Delta r})^2 r\Delta\frac{1}{r}\Delta\Phi_k^{n-1} \quad (5)$$

The solution of these equations, that are in radial direction $\{m\}$ can be presented as

$$a_m\Phi(m+1) + b_m\Phi(m) + c_m\Phi(m-1) - d_m = 0$$

consists in "round trip" calculation

$$\alpha_m = -\frac{a_m}{b_m + c_m\alpha_{m-1}}$$

$$\gamma_m = -\frac{\gamma_{m-1}c_m + d_m}{b_m + c_m\alpha_{m-1}}$$

$$\Phi(m) = \alpha_m\Phi(m+1) + \gamma_m$$

Calculation Time

On one hand it takes two times more "computer time" in comparison with explicit algorithm, but on the other, as the time step is two time larger (usually in explicit algorithm $c\Delta t = 0.5\ \Delta z$), then the implicit method takes approximately the same "computer time" for the same coordinate step. However, as the implicit method does not need great number of mesh points on the bunch length, it has great advantage for short bunch calculation.

B.Zotter comparison

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

O. Meincke, A. Wagner, B. Zotter

SL-Note-97-xx (AP)

New Wake Field and Bunch Lengthening Codes

Abstract

Two computer codes have recently been presented by A. Novokhatski during a short visit to CERN: the first one for calculating longitudinal wake potentials of short bunches in axially symmetric structures from the discretized Maxwell equations, and the other for calculating the evolution of a particle distribution in the presence of wake fields. The first code appears to be much more efficient, and hence faster, than the codes which we have been using at CERN so far. The second one shows intriguing results for bunch shortening by capacitive wakes, and also the development of several peaks in the distribution function for resonator wakes.

Geneva, Switzerland

February 25, 1997

Loss factor comparison

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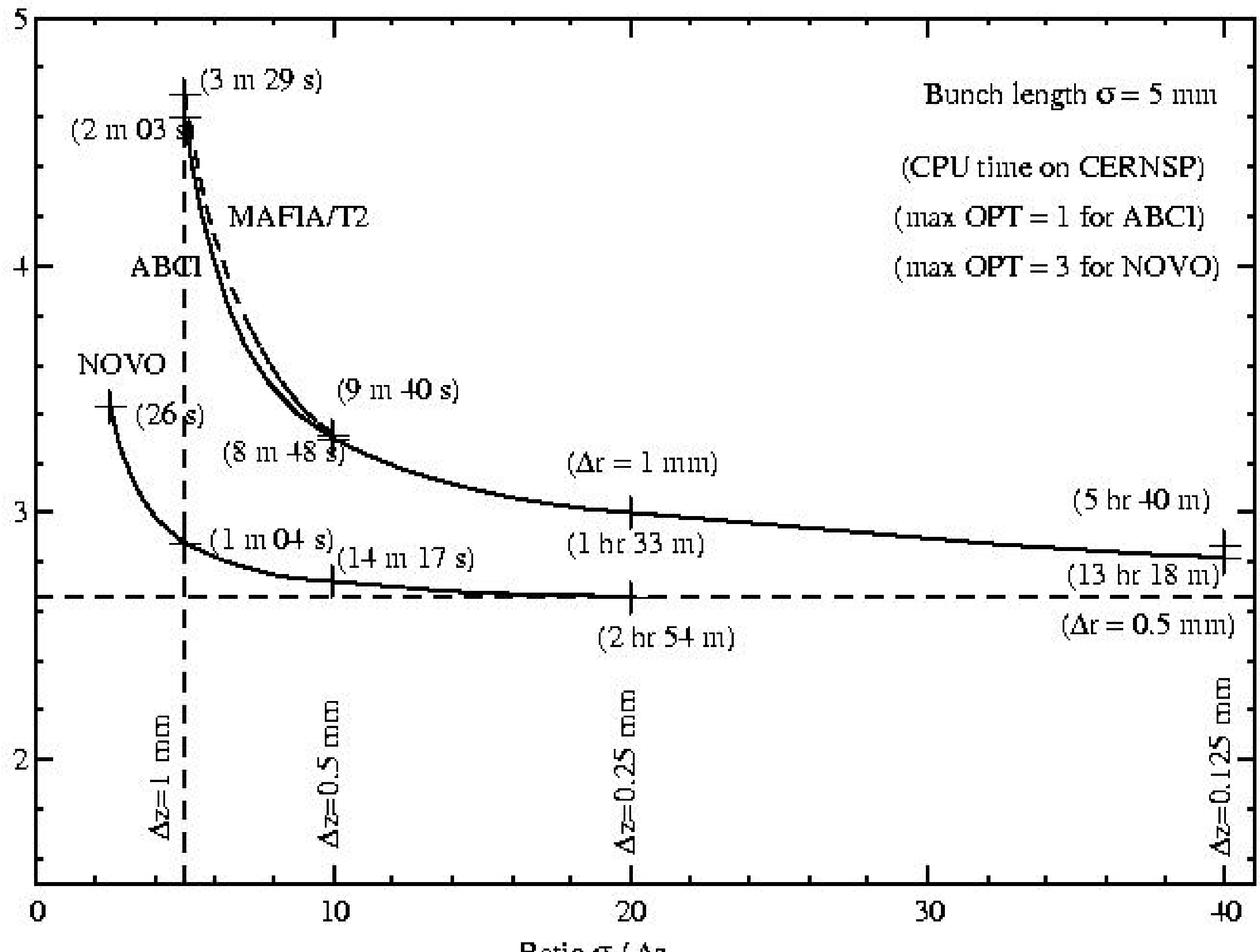
Figure 7: Comparison of Wake potentials computed with NOVO and ABCI



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Wake Potential Comparison

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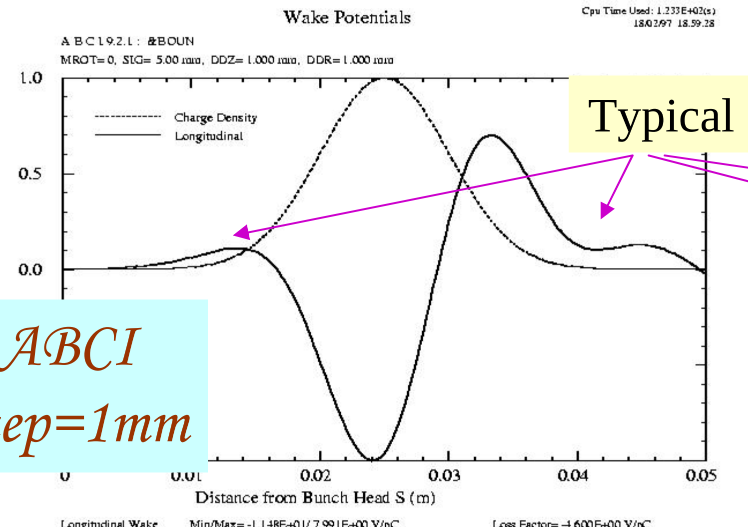
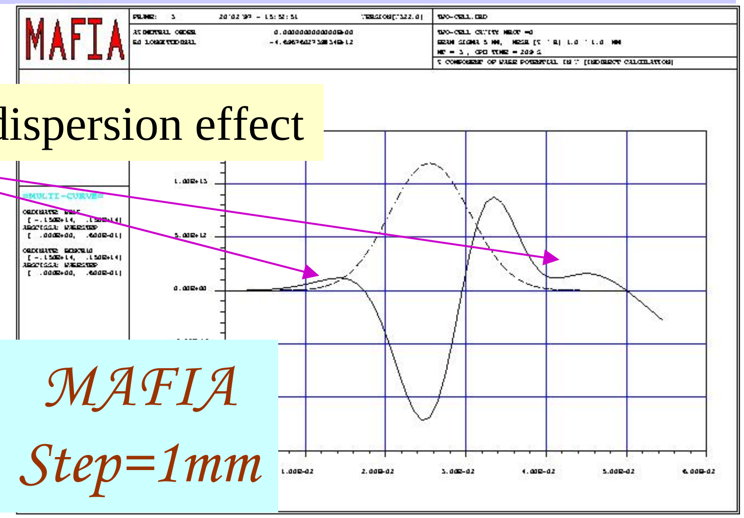
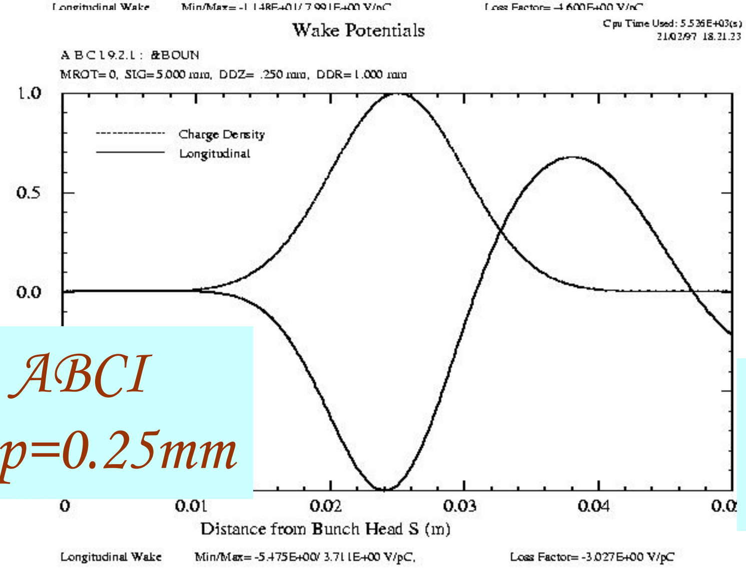
Typical dispersion effect



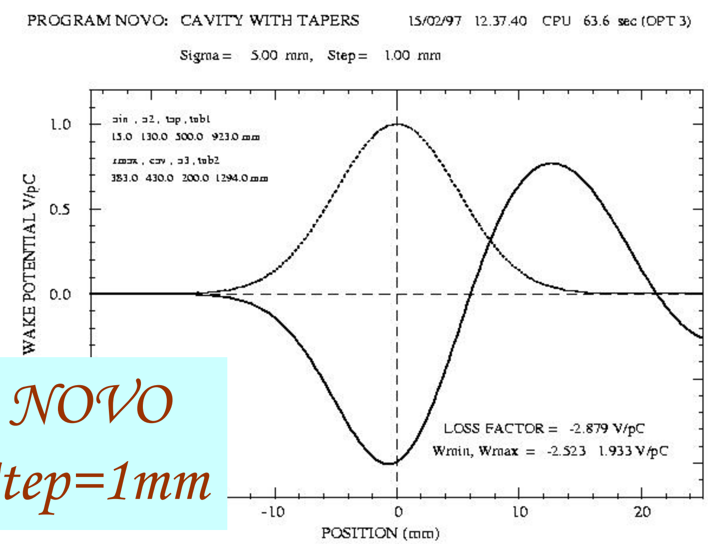
ABCI Step=1mm



MAFIA Step=1mm



ABCI Step=0.25mm



NOVO Step=1mm

RF-cavity for SOLEIL-project

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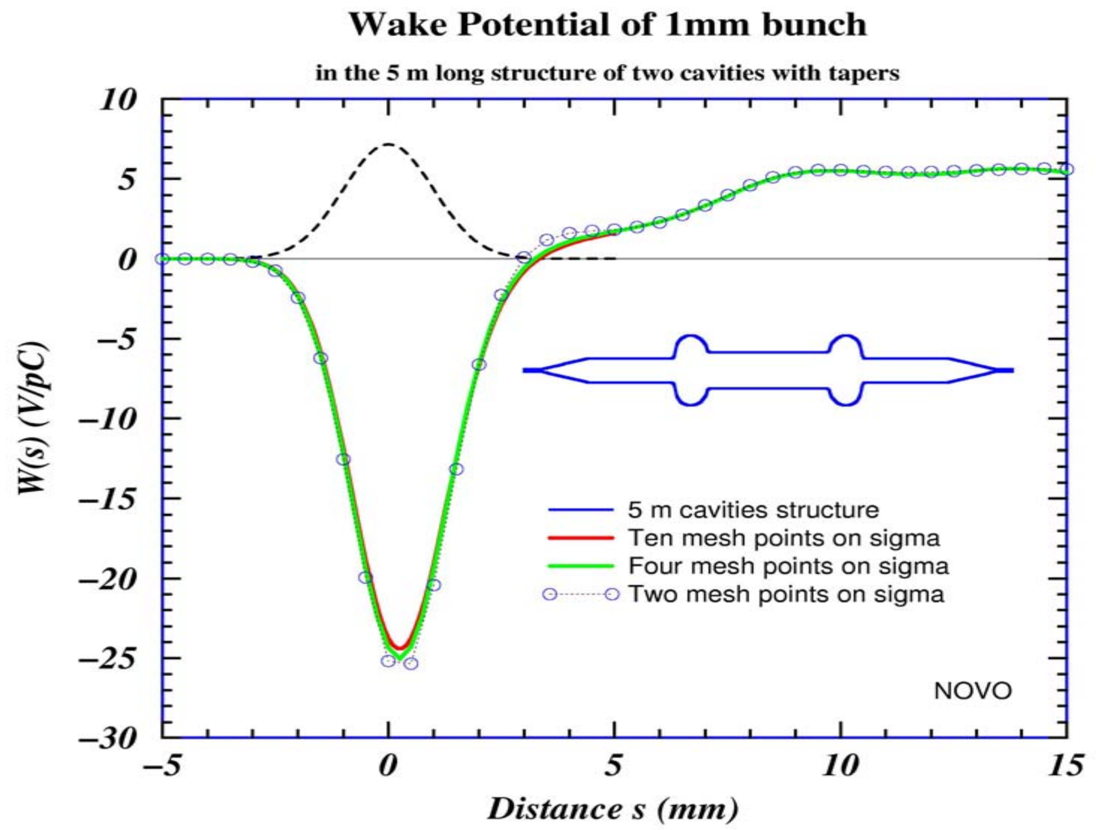

Comparison of the wake fields, calculated for different number of mesh points on the bunch length.

Wake Fields due to Surface Roughness. Simulations.

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Random bumps

<δ>=5 μ

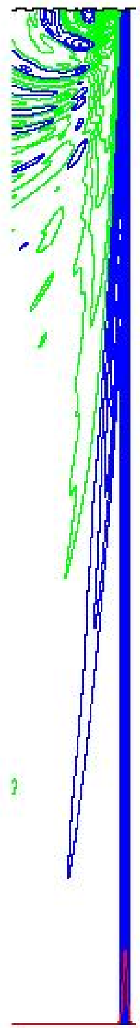
<g>=50 μ

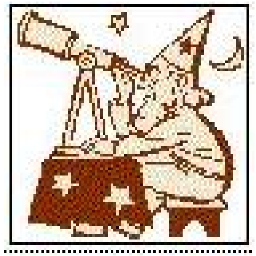
in the tube

R=5mm

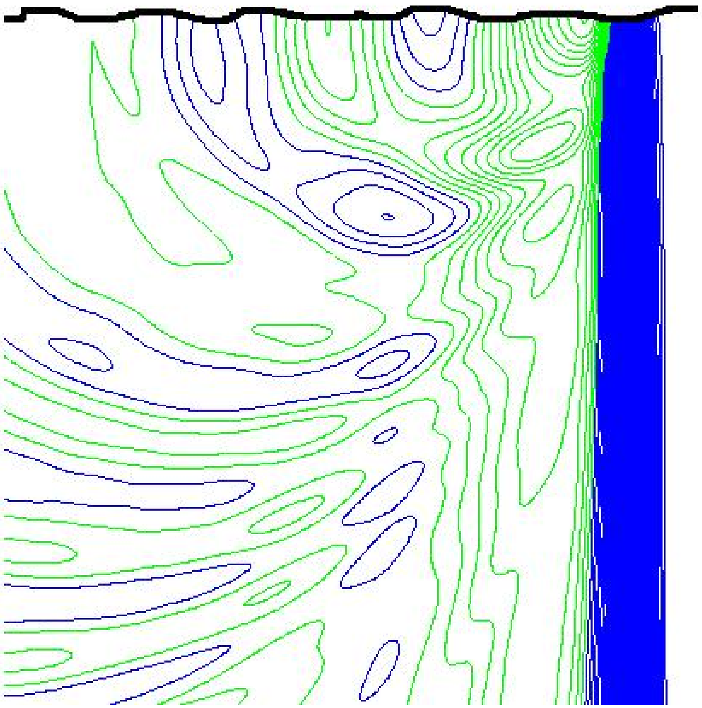
Bunch

σ=10 μ





0.001 ?



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Surface Roughness Wake Fields Chasing a Bunch

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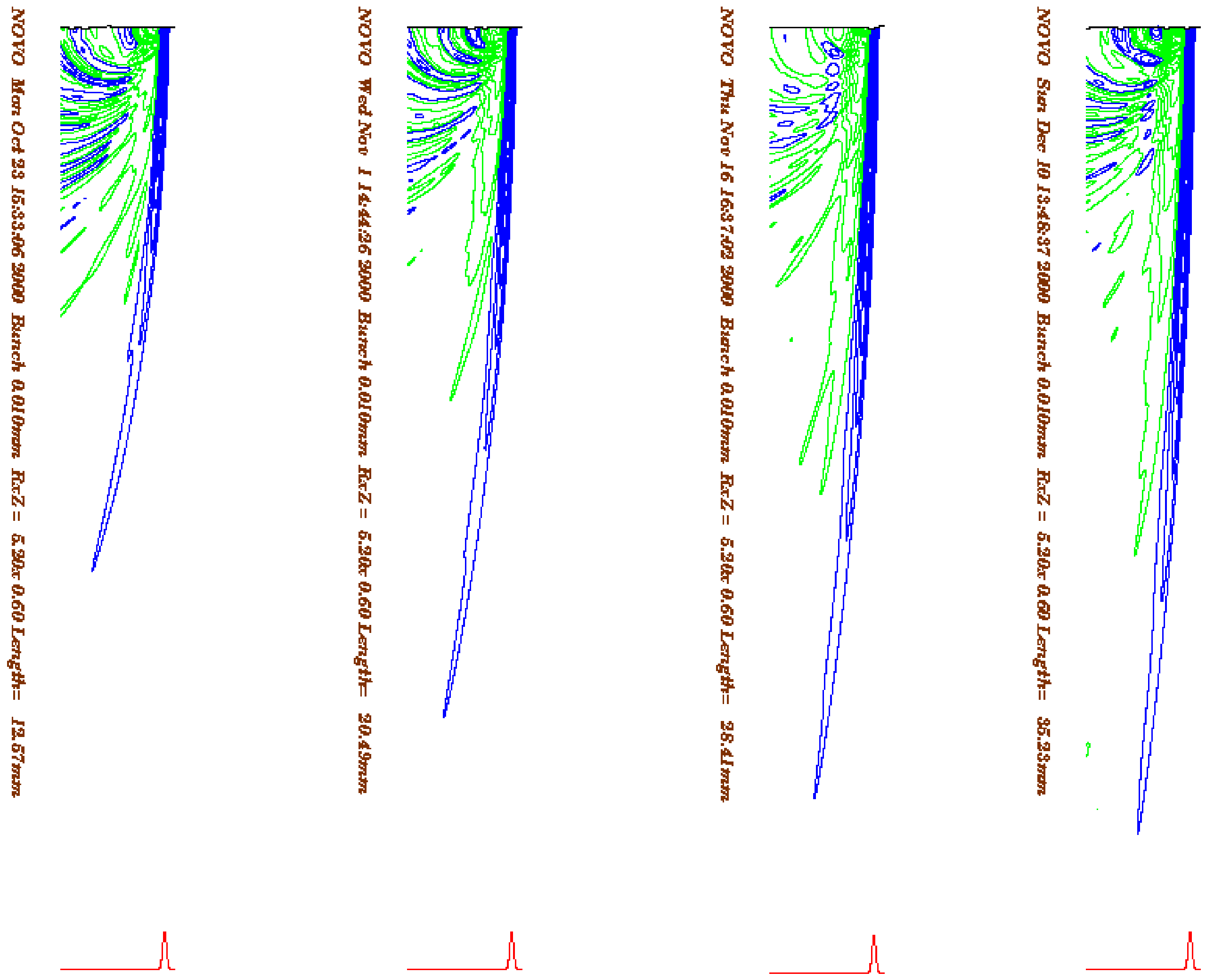
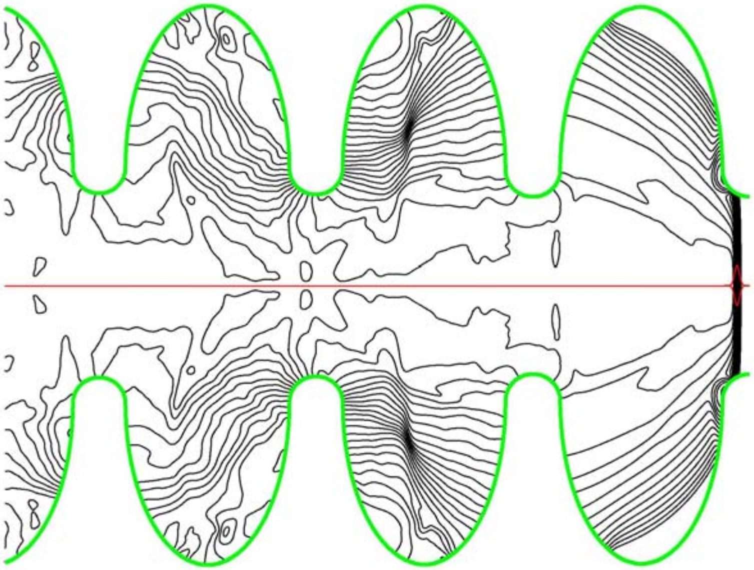


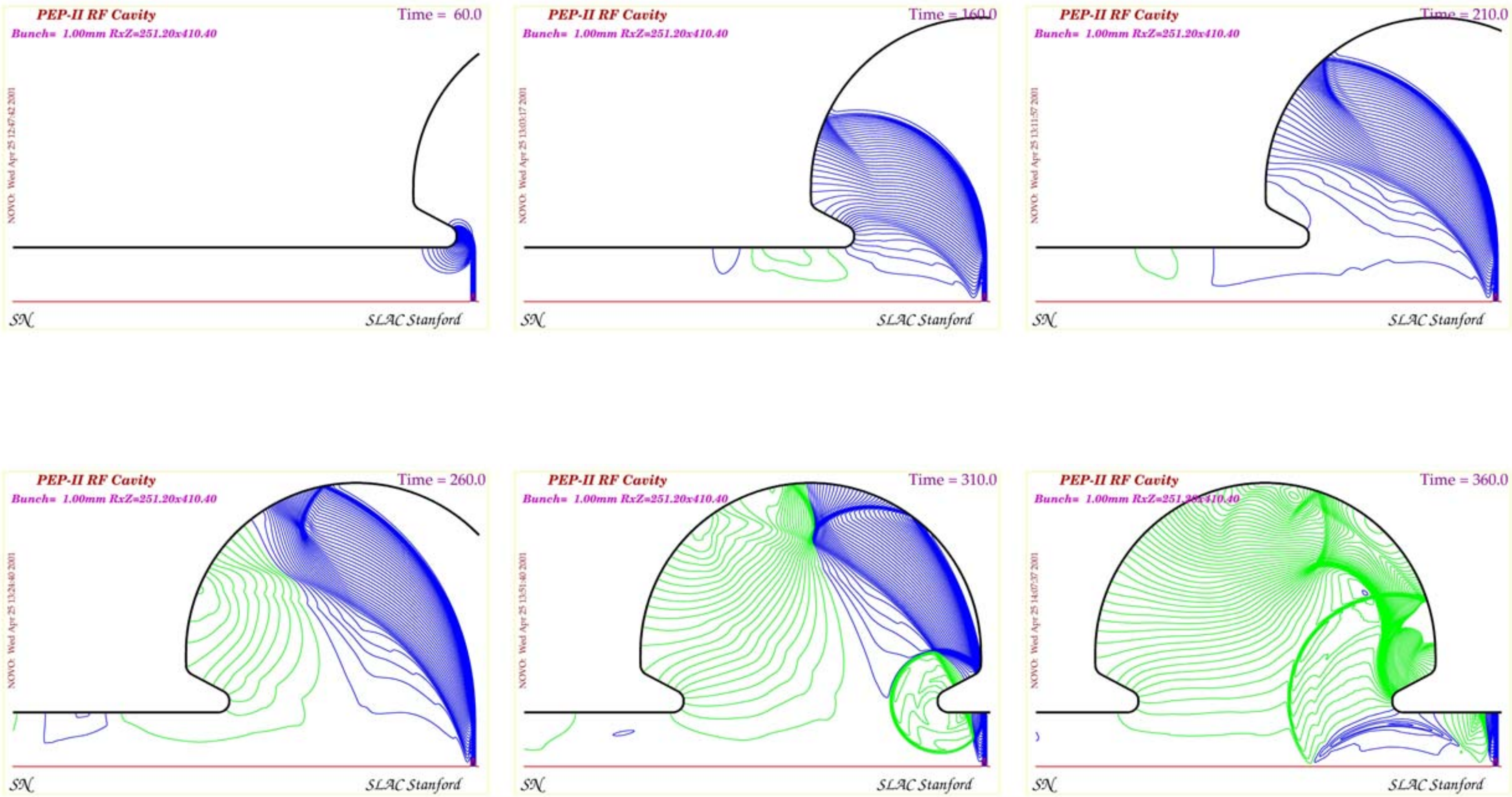
Application to NLC.
Full Accelerating Section.

Electric Force Lines. Bunch is at the end of the NLC Section



100 μ bunch

PEP-II cavity.
Electric force lines.



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