Analysis of FEL Performance Using Brightness Scaled Variables

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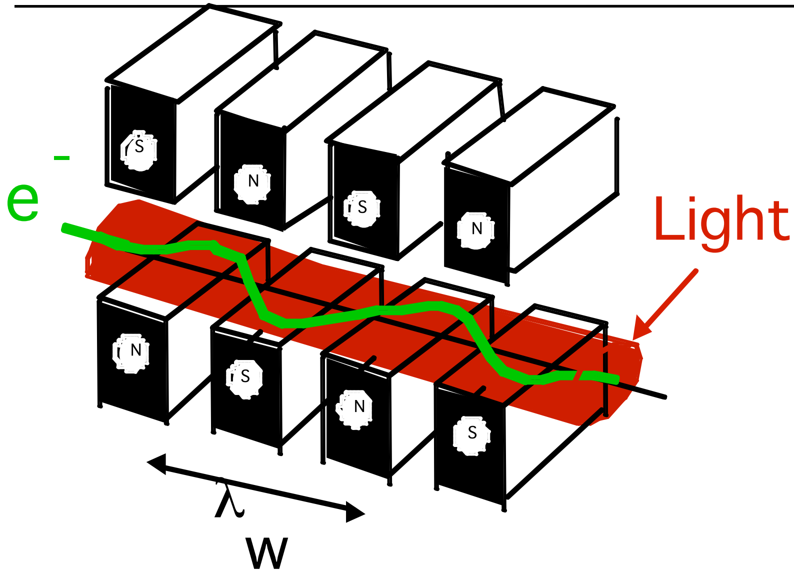
Outline

- Introduce brightness and discuss beam phase space manipulation as a central theme in control of FEL radiation.
  - Focus on ESASE.
- Motivate and define brightness scaling through discussion of linear theory.
- Explore parameter space with fixed brightness beam for an infinite beam and for LCLS with ESASE.
- If there is time, discuss concepts related to flat beams, conditioning, and harmonics for future X-ray FELs.

FEL Basics



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Spread in this term is harmful!

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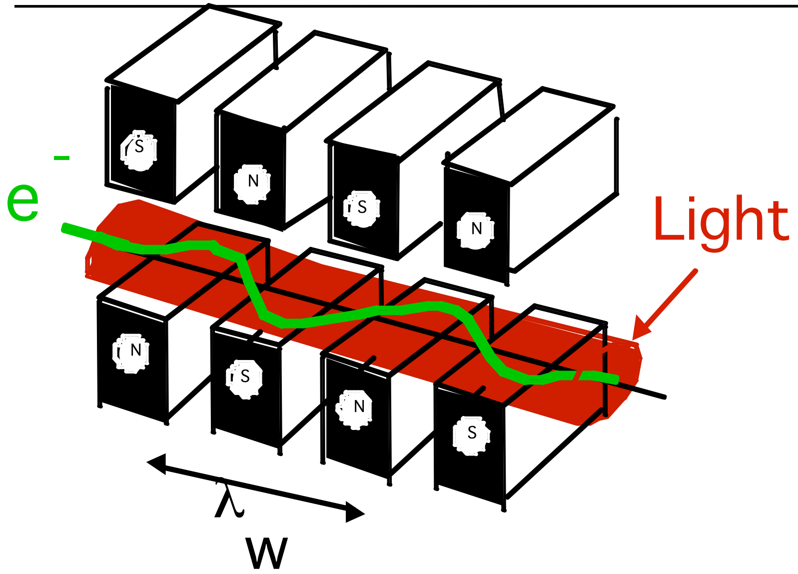

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$$\psi = (k_s + k_w)z - \omega_s t$$

$$v_z = \frac{\omega_s}{k_s + k_w}$$

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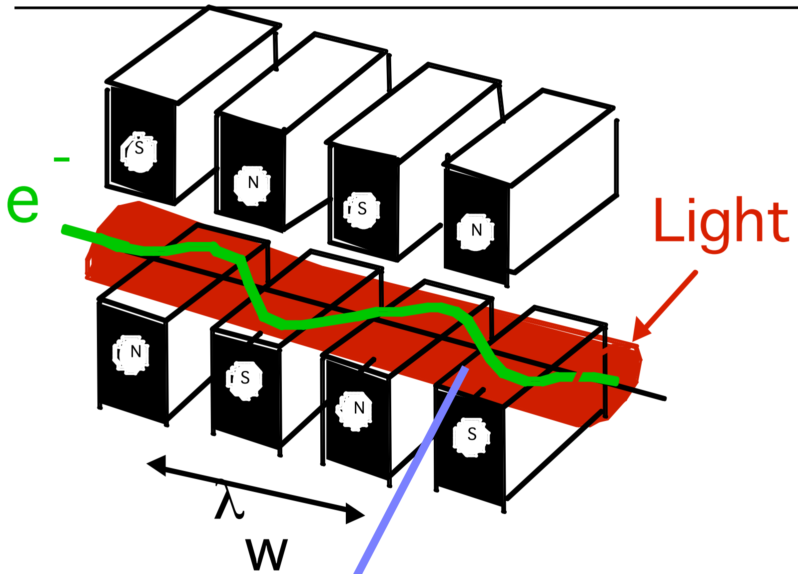

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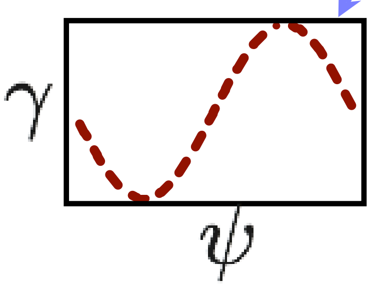

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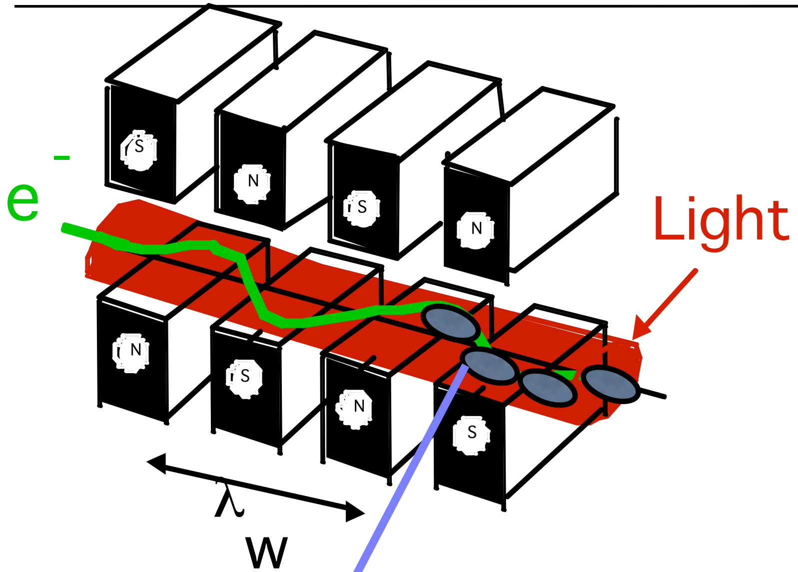

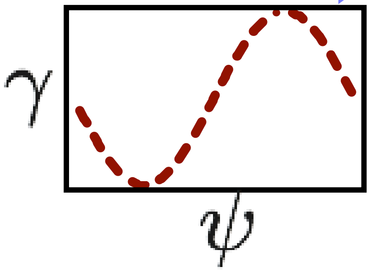

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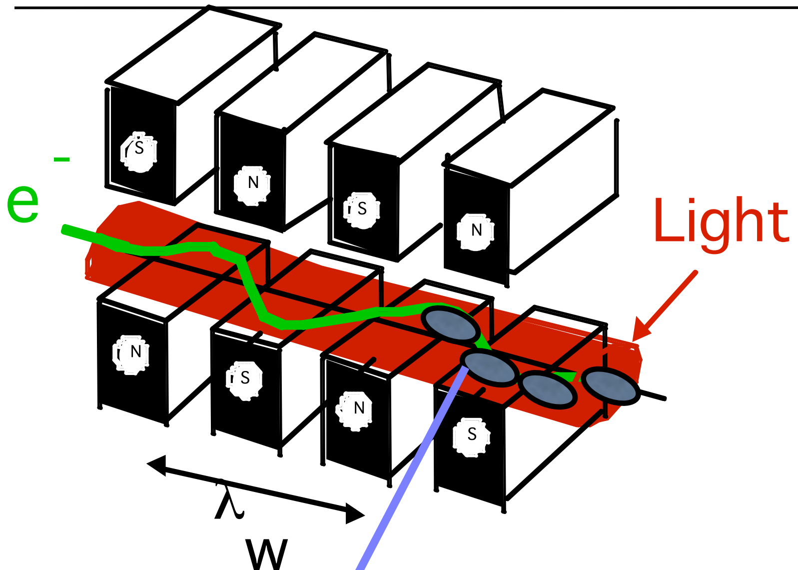

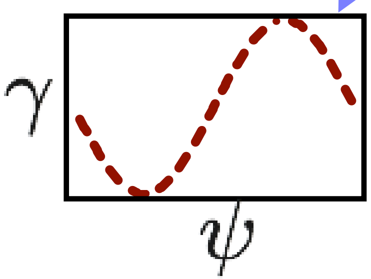

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$$\boxed{\lambda_s = \lambda_w \frac{1+\gamma^2\beta_\perp^2}{2\gamma^2}}$$

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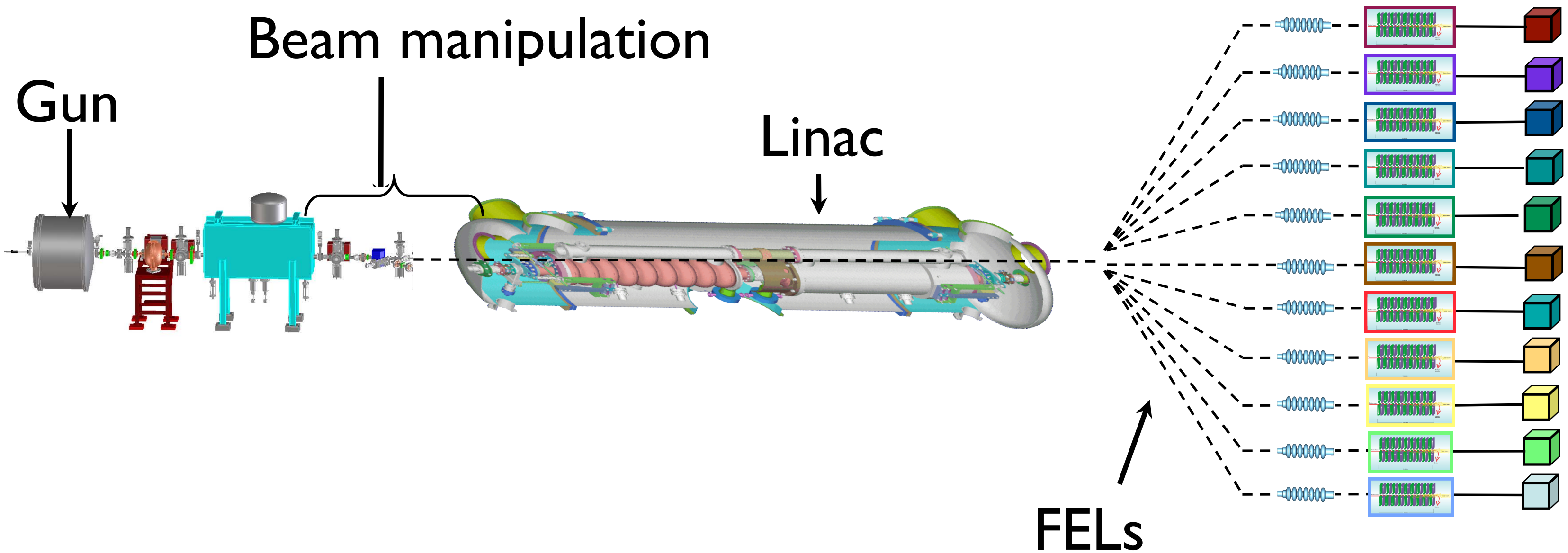
Phase Space Manipulation

Most schemes to improve FEL performance focus on optimizing the beam phase space (at fixed brightness).

- ESASE, emittance exchange, beam conditioning ...

Manipulating beam phase space can have many advantages:

- Enhanced gain (shorter pulses and undulators)
- Seeding radiation pulse for harmonic cascades
- Attosecond pulses
- Synchronization
- Relax beam quality constraints (conditioning)
- Lower energy for given wavelength



FEL performance is governed by beam brightness:

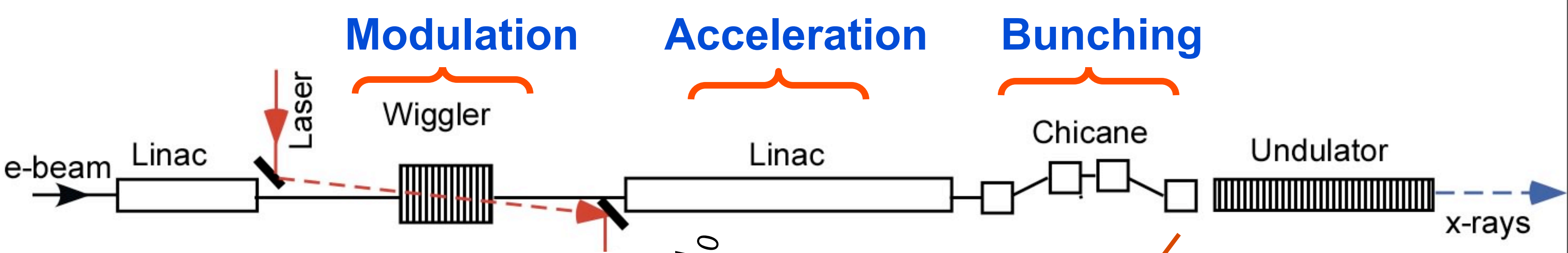
Brightness = # electrons/6D-phase space volume

This number will NOT get larger---

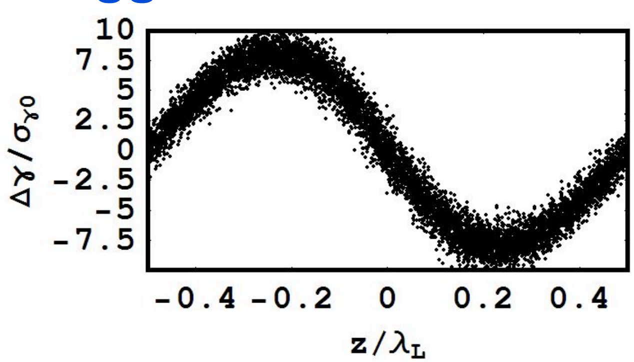
determined by gun physics and emittance

or energy spread can grow through various instabilities

Current-Enhanced SASE (ESASE)

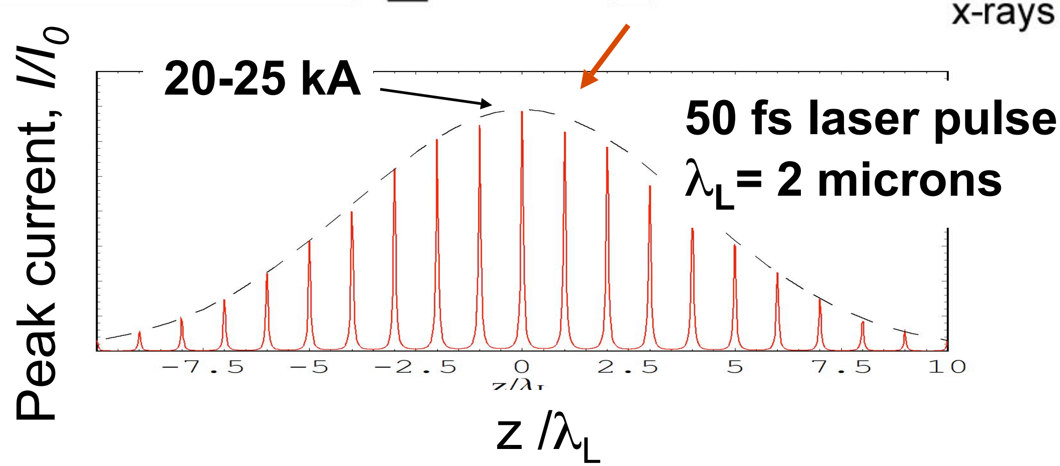


Energy modulation in the wiggler at ~ 4 GeV

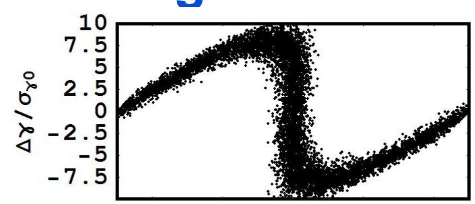


Only one optical cycle is shown

- **Laser peak power ~ 10 GW**
- **Wiggler with ~ 10 periods**



- **Electron beam after bunching at optical wavelength**



(Courtesy A. Zholents)

A. A. Zholents, Phys. Rev. ST Accel. Beams **8**, 040701 (2005)

Brightness Scaling

Just using coupling between longitudinal beam and radiation phase space misses physics.

$$\rho^3 = \frac{e^2 a_1^2 n_e}{16\epsilon_0 \gamma_0^3 m_e c^2 k_u^2}$$

$$\frac{1}{L_{1\mathrm{d}}} = 2\sqrt{3} k_u \rho$$

Instead try and capture the full coupling between the 6D beam and radiation phase space.

$$B_N = \frac{N_e}{\varepsilon_x^n \, \varepsilon_y^n \, \sigma_\gamma \, \sigma_z}$$

$$\frac{1}{L_B} = \frac{1}{2} \frac{a_h^2 \, r_{cl}}{h \, k_1} B_N$$

$$a_h = a_u (-1)^{(h-1)/2} \left(J_{(h-1)/2}(h\xi) - J_{(h+1)/2}(h\xi) \right)$$

Scaling also reflects practical limitations on e-beam production.
For LCLS at .15 nm : L_B=0.51 m, $L_{1\mathrm{d}}$=3.3 m, and L_G=5.1 m

Three-Dimensional Analysis of FEL

- Couple the evolution of the particle distribution to the radiation field through the Maxwell-Vlasov equations for the FEL.
- Arrive at dispersion relation for self-similar modes growing with the distribution function, with growth rate: *q/2L*.

Diffraction Odd harmonic number FEL coupling Coupling strength to odd harmonics

$$\left(\frac{\nabla_\perp^2}{2k_1 h} + \frac{iq_h}{2L}\right)A(\boldsymbol{x}) - 4i\,k_u^3\,\rho^3\left(\frac{a_h}{a_1}\right)^2$$

$$\times \int_{-\infty}^{\infty} \mathrm{d}^2\boldsymbol{p} \int_{-\infty}^{\infty} \mathrm{d}\eta \frac{\partial f_0}{\partial \eta} \int_{-\infty}^{0} \mathrm{d}s\, e^{\Phi s} A(\boldsymbol{x}_\beta) = 0$$

Unperturbed trajectory

$$\eta = (\gamma - \gamma_0)/\gamma_0$$

Frequency detuning from resonance

$$\Phi = q_h/2L - ik_u\Delta\nu_h + ih\Big(2k_u\eta - k_1(J_x/\beta_x + J_y/\beta_y)\Big)$$

$$\left\{\begin{aligned} x_\beta &= x\cos(k_{\beta_x}s) + \frac{p_x}{k_{\beta_x}}\sin(k_{\beta_x}s), \\ y_\beta &= y\cos(k_{\beta_y}s) + \frac{p_y}{k_{\beta_y}}\sin(k_{\beta_y}s), \end{aligned}\right.$$

Beam Conditioning, Energy Spread and Emittance:

$$f_0(\eta, \boldsymbol{J}_\perp) = f_\perp(\boldsymbol{J}_\perp)\, f_\parallel(\eta|\boldsymbol{J}_\perp)$$

Scale Length: L

Z. Huang, K.-J. Kim, Phys. Rev. E **62**, 7295 (2000) and references therein.

Exact Solution

- Assume a gaussian distribution.
- Scaling is chosen to remove the explicit brightness dependence from the equations.
- Solve numerically using a variational approximation for radiation mode.*
- Also includes conditioning, odd harmonics and asymmetric beams

$$B_N = \frac{N_e}{\varepsilon_x^n\,\varepsilon_y^n\,\sigma_\gamma\,\sigma_z} \qquad \frac{1}{L_B} = \frac{1}{2}\frac{a_h^2\,r_{cl}}{h\,k_1}B_N$$

*M. Xie, NIMA, **445**:59-66, (2000).

$$A(\boldsymbol{\xi}) = \int_{-\infty}^{\infty} \mathrm{d}^2\boldsymbol{\xi}'\, T(\boldsymbol{\xi},\boldsymbol{\xi}')A(\boldsymbol{\xi}')$$

$$T(\boldsymbol{\xi},\boldsymbol{\xi}') = \int_{-\infty}^{0} \frac{4h^2\, b_\gamma \sqrt{b_{\varepsilon_x} b_{\varepsilon_y}}\,\tau\mathrm{d}\tau\, e^{(q_h - i b_\omega)\tau - (2hb_\gamma)^2\tau^2/2}}{i q_h - \frac{2b_{d_x}}{h}\xi_x^2 - \frac{2b_{d_y}}{h}\xi_y^2} \times \prod_{\ell=x,y} \frac{\exp\left(-\frac{\xi_\ell^2 + {\xi'}_\ell^2 - 2\xi_\ell {\xi'}_\ell \cos(2\sqrt{b_{d_\ell} b_{\varepsilon_\ell}}\tau)}{2(1+ihb_{\varepsilon_\ell}(1-\bar{\kappa}_\ell)\tau)}\right)}{\sqrt{2\pi}(1+ihb_{\varepsilon_\ell}(1-\bar{\kappa}_\ell)\tau)}.$$

Diffraction	Angular Spread
$b_d = \frac{L_B}{2k_r\sigma_x^2}$	$b_\varepsilon = 2L_B k_1 \frac{\varepsilon}{\beta}$
$b_\gamma = 2L_B k_u \sigma_\eta$ **Energy Spread**	$b_\omega = 2L_B k_u \Delta\nu$ **Detuning**

Fundamental scaling parameters for FEL.

The Infinite Beam

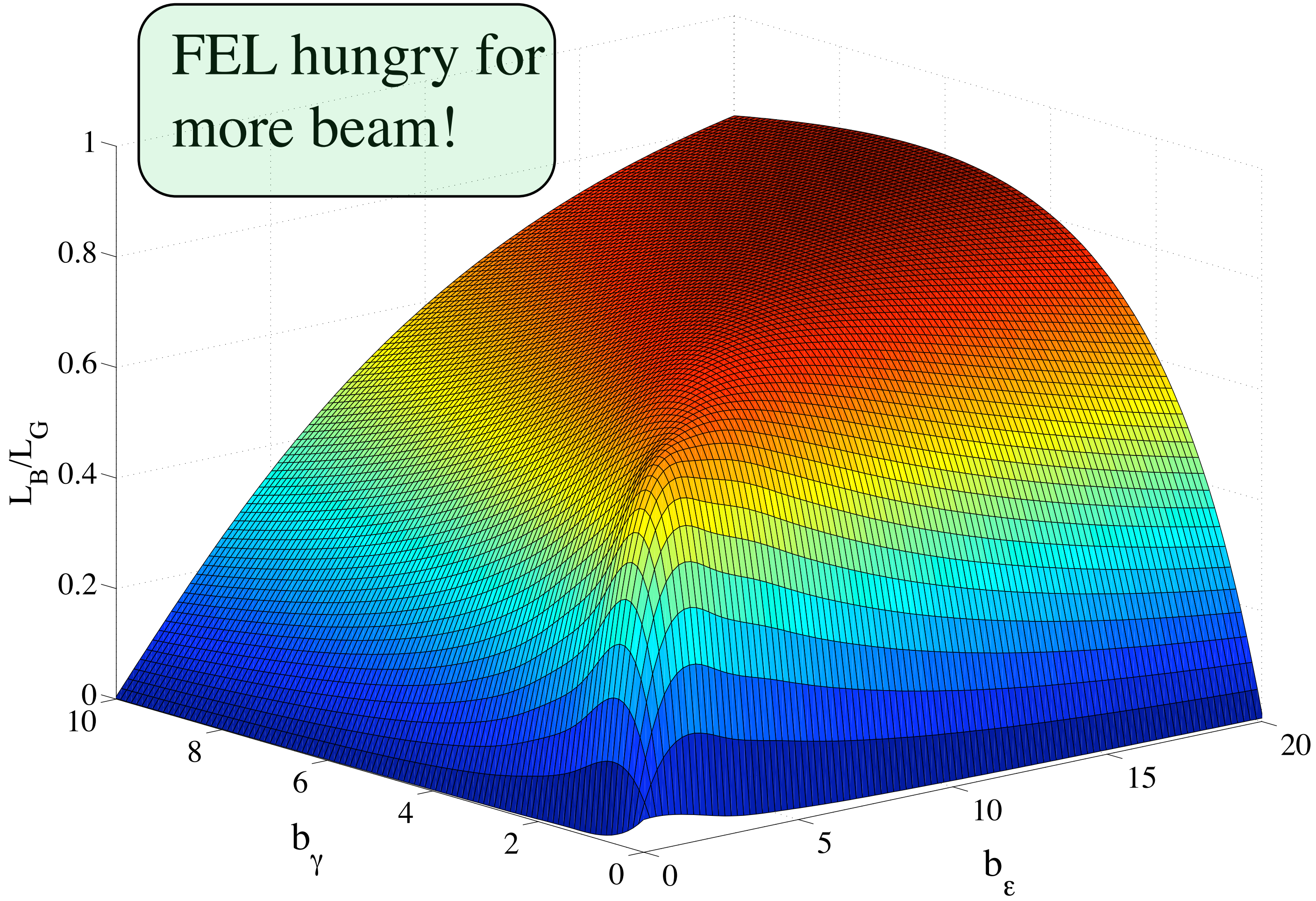
- By taking an infinite beam we remove diffractive effects.
- Transverse modes become degenerate with growth rate:

$$q + 2i\left(2hb_\gamma/hb_\varepsilon\right)\int_{-\infty}^{0}\frac{z_b \mathrm{d}z_b \; e^{(q/hb_\varepsilon - ib_\omega/hb_\varepsilon)z_b - (2hb_\gamma/hb_\varepsilon)^2 z_b^2/2}}{1 + i(1-\bar{\kappa})z_b} = 0$$

Here h is the harmonic number, and z_b is scaled to $2L_B/hb_\varepsilon$

- Note that the harmonic number can be scaled out of the equations.
- Can take parameters to infinity to simplify equation and obtain convenient relations.

Infinite Beam Parameter Space



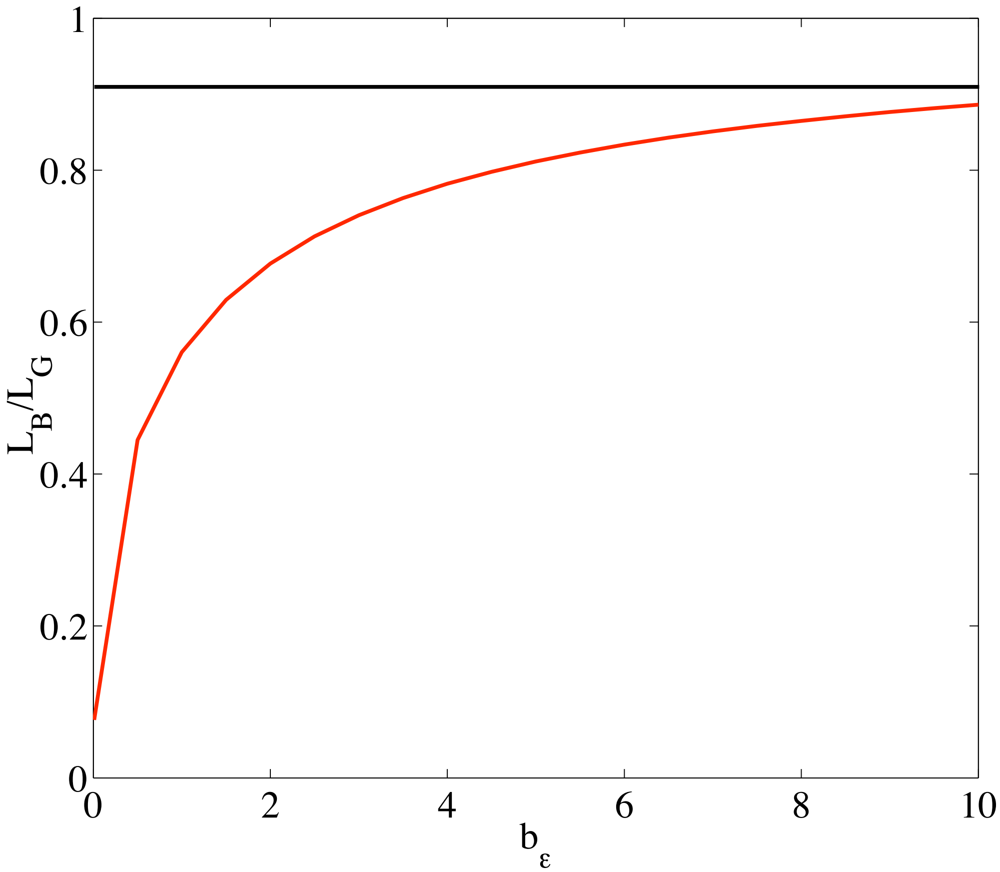
Optimal Phase Space

- Every value of the angular spread has optimum energy spread and detuning.
- In asymptotic region we find:

$$k_u\,\sigma_\gamma/\gamma = 0.3\,k_1\,\varepsilon/\beta$$

$$k_u\,\Delta\omega/\omega_1 = -1.3\,k_1\,\varepsilon/\beta$$

- For LCLS parameters this implies $\beta \sim 25$ m
- All deleterious effects on odd harmonics are captured in L_B.



The Phase Equation

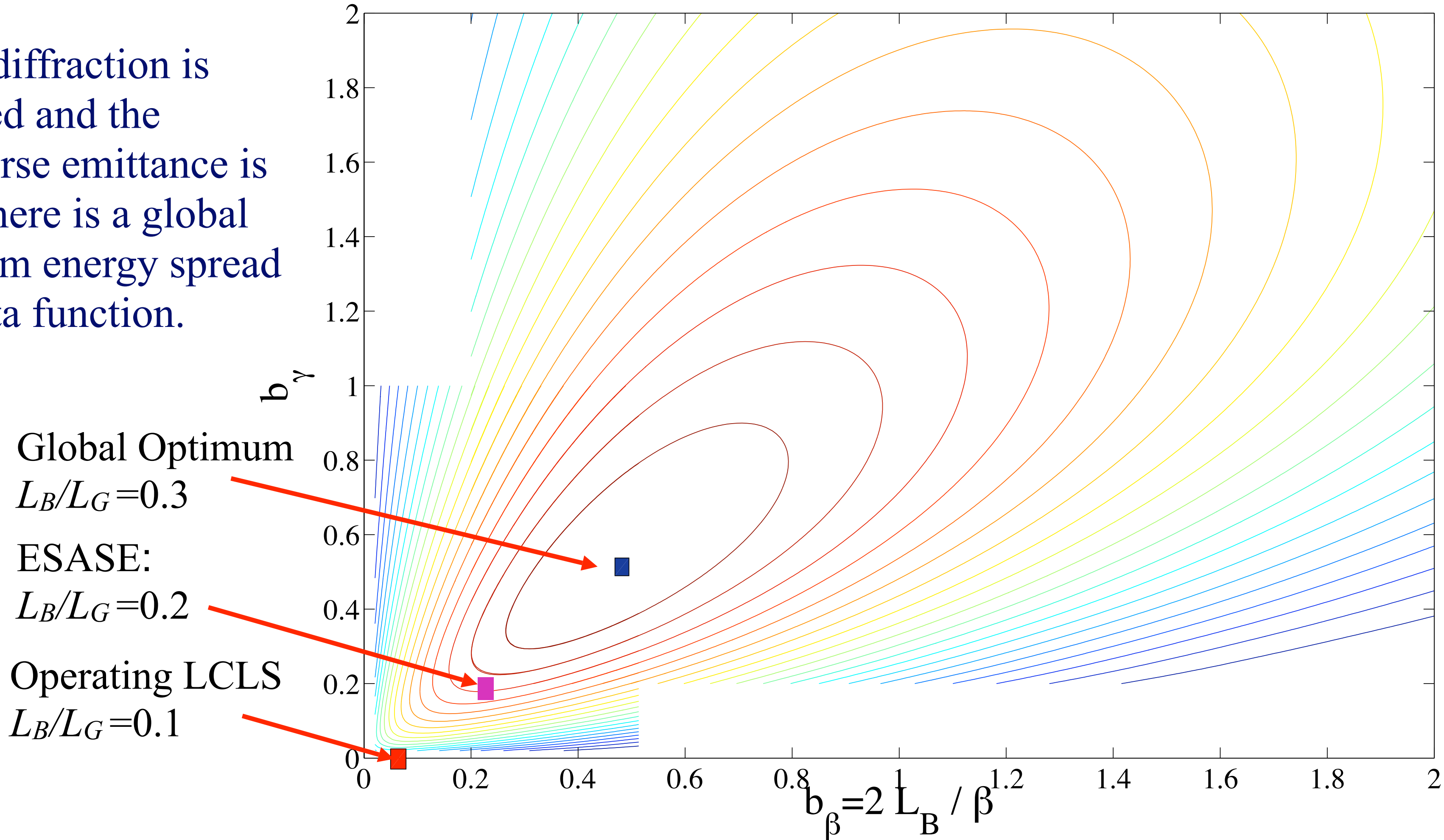
- The phase of the electrons in the combined wiggler and radiation field is governed by:

$$\psi' = 2k_u \frac{\Delta\gamma}{\gamma_0} - \frac{k_r}{\beta}(J_x + J_y)$$

- Increasing phase space volume at fixed brightness adds enough particles to the “conditioned” part of phase space to sustain growth.

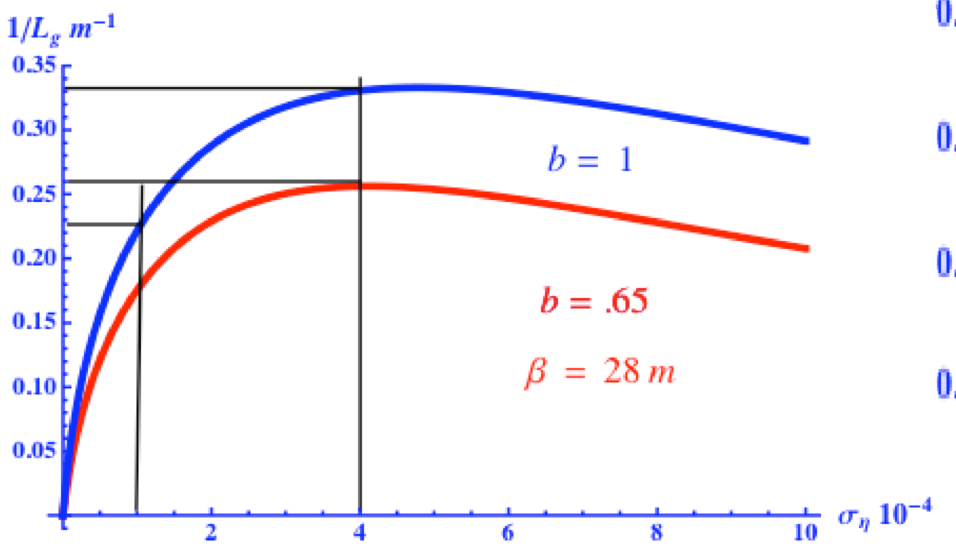
LCLS with ESASE

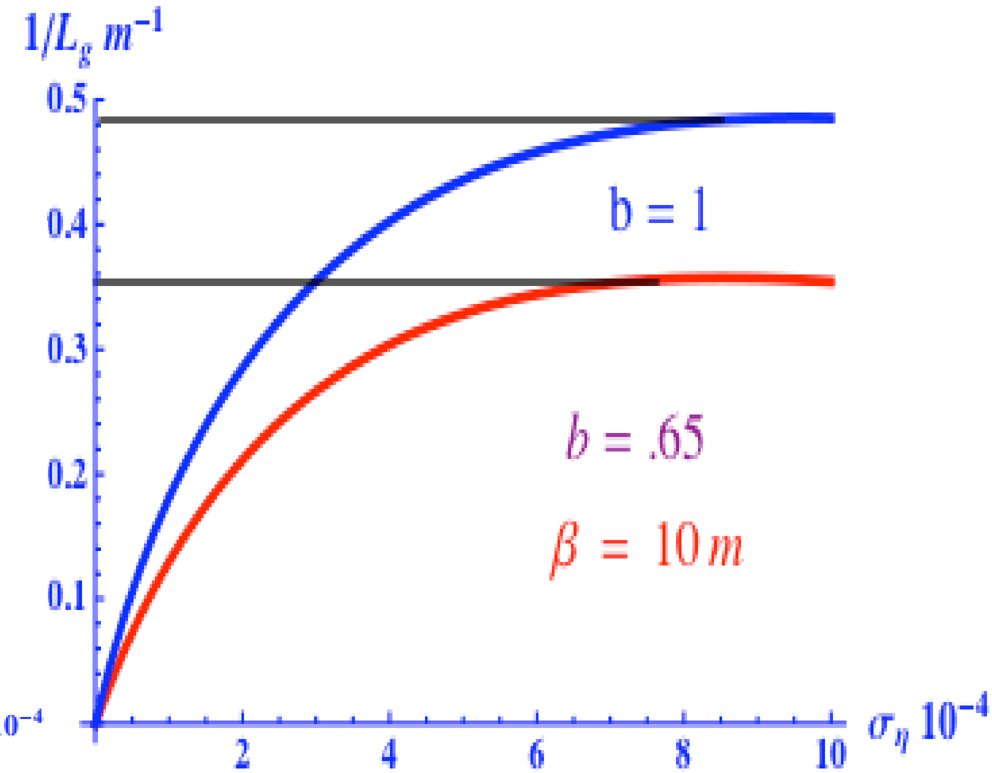
When diffraction is included and the transverse emittance is fixed there is a global optimum energy spread and beta function.



ESASE Performance

If ESASE does not conserve brightness must shorten the beta function





Conclusions

$$B_N = \frac{N_e}{\varepsilon_x^n \, \varepsilon_y^n \, \sigma_\gamma \, \sigma_z} \qquad \frac{1}{L_B} = \frac{1}{2} \frac{a_h^2 \, r_{cl}}{h \, k_1} B_N$$

- Phase space manipulation is a powerful tool for controlling FEL output.
- Brightness scaling in the linear theory allows a much simpler analysis of FEL optimization that reflects realistic constraints in beam physics and technology.
- This work suggests novel strategies for the generation of very short wavelengths or high-intensity FEL output.

References

M. Xie, Nucl. Instrum. Methods Phys. Res., Sect. A, **445**:59-66, (2000).

M. Xie, Nucl. Instrum. Methods Phys. Res., Sect. A, **507**:450-454, (2003).

Z. Huang, K.-J. Kim, Phys. Rev. E **62**, 7295 (2000).

A.M. Sessler, D.H. Whittum, L.-H Yu, Phys. Rev. Lett. **68**, 309 (1992).

A. Wolski, *et al.*, Phys. Rev. ST Accel. Beams **7**, 080701 (2004).

A. A. Zholents, Phys. Rev. ST Accel. Beams 8, 040701 (2005)