

DRAFT
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Review of Feedback Basics

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1 Abstract

Basics of the feedback theory are presented.

2 Plan:

- Basic feedback:

closed and open loop transfer functions,

Nyquist criterion of stability.

- Longitudinal feedback:

cavity impedance,

steady state conditions, optimum conditions,

F.Pedersen's analysis

- Actual longitudinal feedbacks
- Transverse feedback.

3 Transfer functions of control system

Dynamics of a linear system is described by a system of linear diff. equations $\hat{L}_n(t)y(t) = U(t)$ of the n-th order,

$$\sum_{k=0}^n a_k \frac{d^k y(t)}{dt^k} = \sum b_k \frac{d^k u(t)}{dt^k},$$

where a_k, b_k are constants.

Laplace transform $y(t) \rightarrow \tilde{y}(s)$, $y(s) = \int_0^\infty f(t)e^{-st}$,

$$y(t) = \int_{\sigma-i\infty}^{\sigma+i\infty} \frac{ds}{2\pi i} \tilde{y}(s)e^{st},$$

gives for driven solution $\tilde{y}(s) = G_n(s)\tilde{u}(s)$, where $G_n(s)$ is *the transfer function*,

$$G(s) = \frac{\sum b_k s^k}{\sum_{k=0}^n a_k s^k} = |G(s)|e^{i\Phi(s)},$$

and the denominator is *characteristic polynomial*.

Single element: $T(s)$

Cascade (open loop transfer function):

$$a \rightarrow b = Ga; b \rightarrow c = Hb = GHa; T = c/a = GH;$$

Parallel elements: $T(s) = T_1(s) + T_2(s)$.

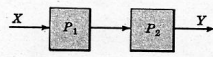
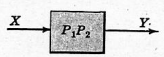
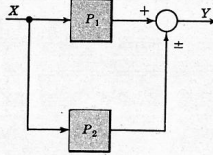
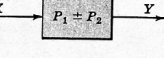
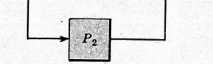
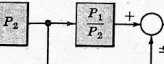
	Transformation	Equation	Block Diagram	Equivalent Block Diagram
1	Combining Blocks in Cascade	$Y = (P_1 P_2)X$		
2	Combining Blocks in Parallel; or Eliminating a Forward Loop	$Y = P_1 X \pm P_2 X$		
3	Removing a Block from a Forward Path	$Y = P_1 X \pm P_2 X$		

Figure 1: Transfer function of a control system.

Feedback transfer function:

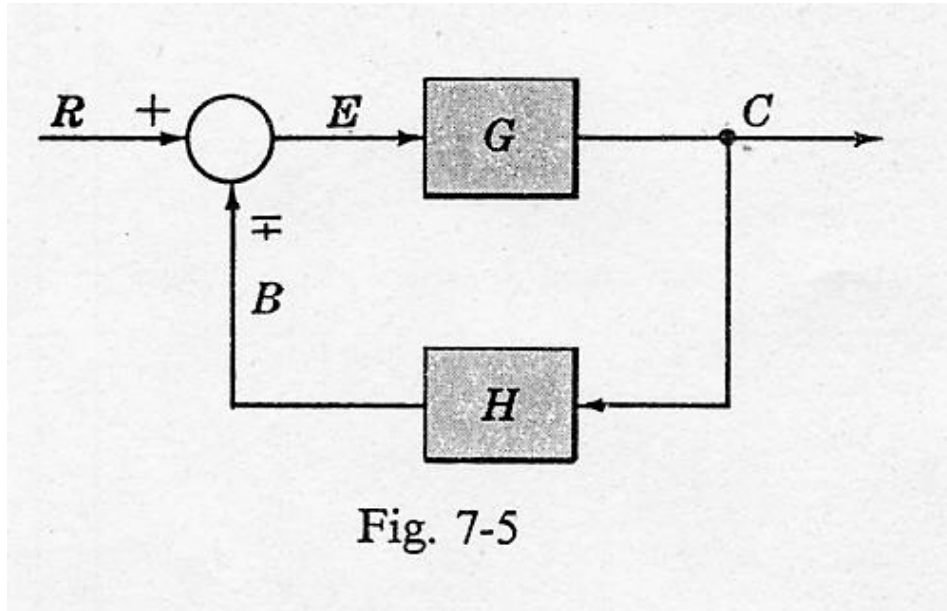


Figure 2: Schematic of the direct FB.

$$E = R - B; C = GE; B = HC = HGE;$$

$$E = R - HGE; E = \frac{R}{1 + HG}; T = \frac{C}{R} = \frac{G}{1 + HG}$$

Note: charact. polinomial GH is the open loop TF.

Note: Discrete feedback system:

$$y_n + by_{n-1} = a_0x_n + a_1x_{n-1}.$$

$b = 0$, finite impulse response filter (FIR),

$b \neq 0$, infinite response filter (IIR). Stable if $|b| < 1$.

3.1 Conditions of stability

The characteristic polynomial is

$$P_n(s) = \sum_{k=0}^n a_k s^k.$$

The eigen-values s_k , $k = 0, 1, \dots, n$ are the roots $P_n(s) = 0$.

Condition of stability $Re[s_k] < 0$.

Routh and Hurwitz conditions of stability: rules on combinations of coefficients a_k .

Nyquist stability criterion:

(don't confuse with the Nyquist criterion of sampling: sampling rate $\frac{1}{\Delta t} > 2\Delta f$):

Consider the transfer function in the form:

$T(s) = \frac{1}{1+GH(s)}$, where $GH(s)$ is open loop transfer function.

Use theorem: for $f(s) = 1 + GH(s)$, the integral

$$J = \frac{1}{2\pi i} \int_C \frac{f'(s)}{f(s)} ds;$$
$$J = P - Z = n_f(0) = n_{GH}(-1)$$

where P and Z are the number of poles/zeros within the contour C in s -plane, and $n_f(0)$ number of loops around the origin by the map $s \rightarrow f(s) = 1 + GH(s)$, or, the number of loops around -1 by $GH(s)$ in $Re[GH], Im[GH]$ plane.

Criterion:

If $n_{GH}(-1) = P$, there are no zeros of $f(s)$, i.e. no poles of $T(s)$ with $Re(s) > 0$ and the system is stable.

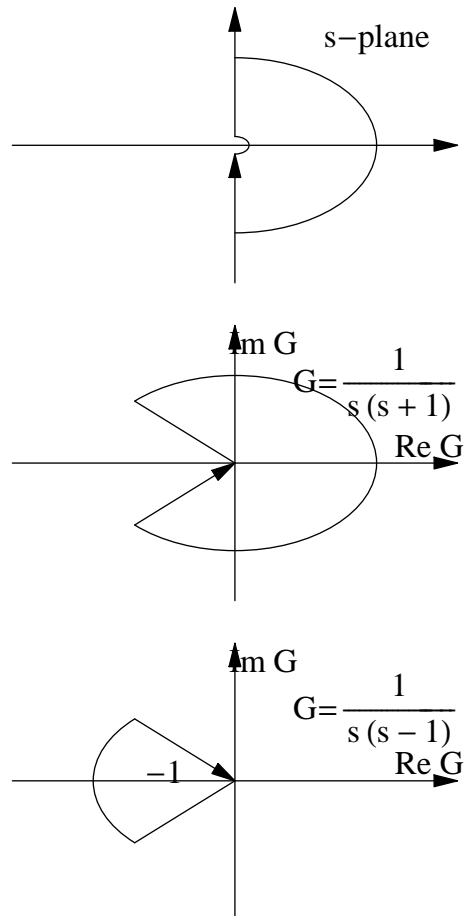


Figure 3: Nyquist criterion for transfer function $G(s)$: upper plot: $n = P = 0$, stable. Lower plot: $n = -1$, $P = 1$, unstable.

4 Low level feedback

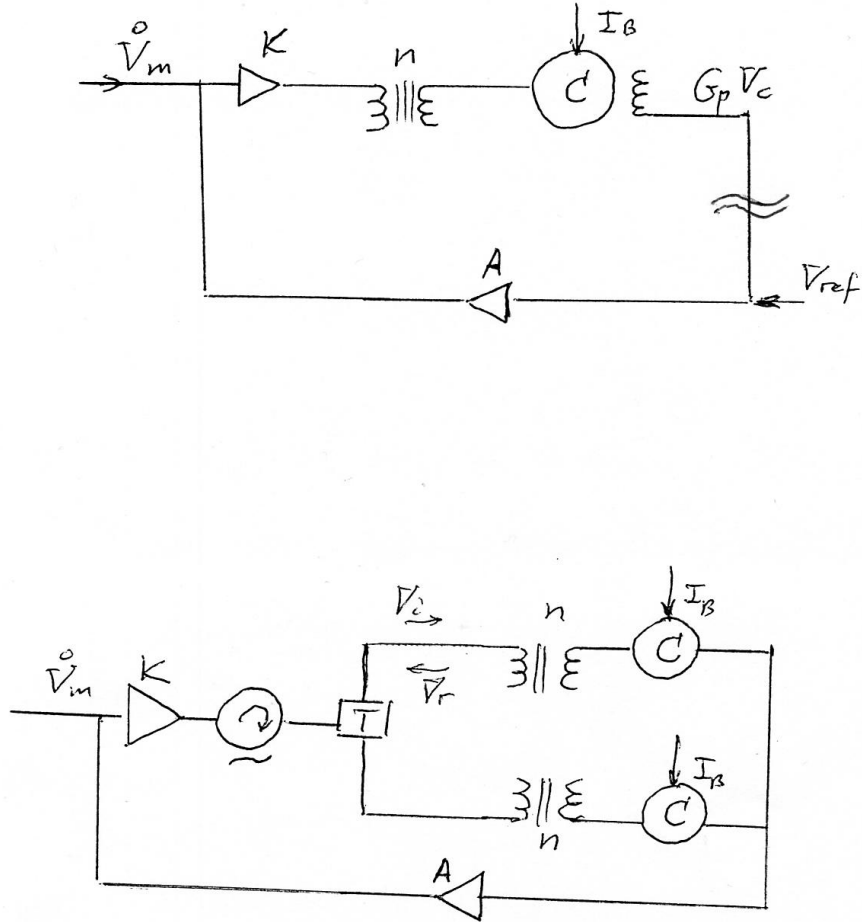


Figure 4: Basic direct loop of the cavity FB.

$$V_c = Z_c(I_g - I_{FB}), \quad I_{FB} = \frac{V_c}{Z_A} = \frac{Z_c}{Z_A}(I_g - I_{FB}),$$

$$I_{FB} = \frac{Z_c}{Z_A} \frac{I_g}{1 + Z_g/Z_A}, \quad V_c = \frac{I_g Z_c}{1 + Z_c/Z_A}$$

4.1 Cavity impedance

Impedance of a cavity including coupling $\propto \beta$ is given in terms of the loaded shunt impedance R_L and loaded Q_L -factor:

$$Z_c(\omega) = \frac{R_L}{1 + iQ_L(\omega_c/\omega - \omega/\omega_c)} \simeq \frac{R_L}{1 - 2iQ_L \frac{\omega - \omega_c}{\omega_c}},$$
$$R_L = \frac{R_0}{1 + \beta}, \quad R_L = \frac{R_0}{1 + \beta}, \quad Q_L = \frac{Q_0}{1 + \beta}.$$

At $\omega = \omega_g$,

$$Z_c(\omega_g) = R_L \cos \psi e^{i\psi},$$
$$\tan \psi = Q_L \left(\frac{\omega_g}{\omega_c} - \frac{\omega_c}{\omega_g} \right).$$

4.2 Low-level FB transfer function

The effective impedance of the cavity with the direct FB:

$$V_c(\omega) = \frac{I_g Z_c}{1 + Z_c/Z_A} = \frac{I_g R_L}{1 - 2iQ_L \frac{(\omega - \omega_c)}{\omega_c} + G_{FB}(\omega)}.$$

$$\begin{aligned} G_{FB}(\omega) &= \frac{R_L}{Z_A} = H(\omega) e^{-i\Phi(\omega)} \simeq H(\omega_c) e^{-i(\omega - \omega_c)\tau_d}, \\ &\simeq H_g (1 - i(\omega - \omega_c)\tau_d) \end{aligned}$$

Hence,

$$Z_{eff} = \frac{R_H}{1 - 2iQ_H(\omega - \omega_c)/\omega_c},$$
$$R_H = \frac{R_L}{1 + H_g}, \quad Q_H = \frac{Q_L}{1 + H_g} + \frac{H_g}{1 + H_g} \frac{\omega_g \tau_d}{2}.$$

FB reduces R_L but increases the bandwidth of revolution harmonics affected by the cavity impedance.

Numbers: $Q_0 = 3.0 \cdot 10^4$, $Q_L = 6.6 \cdot 10^3$, $H_g \simeq 12$,
 $\tau_d = (380 + 150) \text{ ns}$, $\omega_g \tau_d / 2 \simeq 700$.

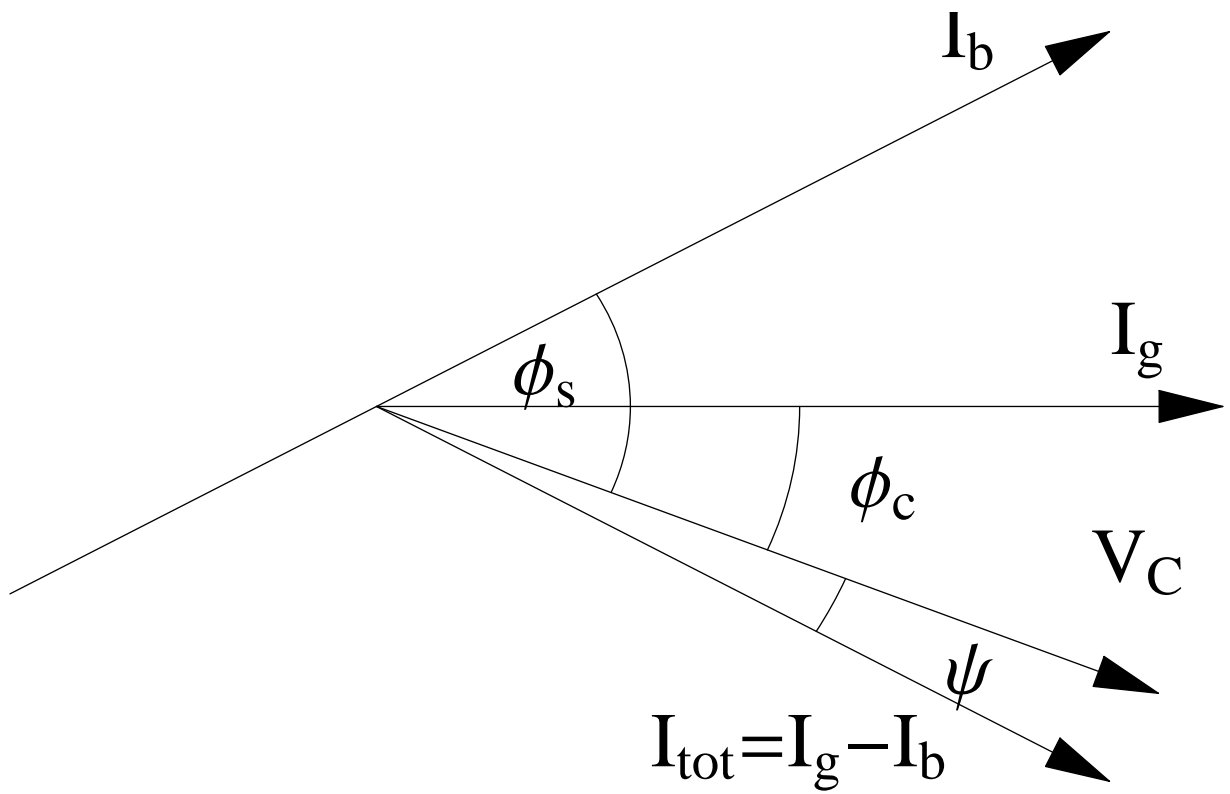


Figure 5: Phase notations.

4.3 Steady state of rf cavity: $\omega = \omega_g$, FB off

Total excitation current for a cavity: $I_{tot} = I_g - I_B$.

Define:

$$I_{beam}(t) = 2I_{dc} \cos(\omega_g t + \phi_c - \phi_s) = \frac{1}{2}(\hat{I}_b e^{-i\omega_g t} + c.c.),$$

$$\hat{I}_b = 2I_{dc} e^{i(\phi_s - \phi_c)}, \quad \hat{I}_b = I_{beam}(\omega_g),$$

$$V(t) = V_{cav} \cos(\omega_g t + \phi_c) = \frac{1}{2}(\hat{V}_c e^{-i\omega_g t} + c.c.),$$

$$\hat{V}_c = V_{cav} e^{-i\phi_c}, \quad \hat{V}_c = V_{cav}(\omega_g)$$

$$I_{gen}(t) = I_g \cos(\omega_g t) = \frac{1}{2}(\hat{I}_g e^{-i\omega_g t} + c.c.),$$

$$\hat{I}_g = I_g, \quad \hat{I}_g = I_{gen}(\omega_g).$$

I_{dc} is the average beam current, V_{cav} is maximum cavity voltage.

Then, defining $\hat{I}_{tot} = \frac{1}{2}(\hat{I}_{tot} e^{-i\omega_g t} + c.c.)$,

$$\hat{I}_{tot} = \hat{I}_g - |\hat{I}_b| e^{i(\phi_s - \phi_c)},$$

$$V_{cav} e^{-i\phi_c} = \hat{I}_{tot} R_L \cos \psi e^{i\psi}.$$

Separating real and imaginary parts,

$$\hat{I}_g = |\hat{I}_B| \frac{\sin(\phi_s + \psi)}{\sin(\phi_c + \psi)}$$

$$\frac{1 + \beta}{Y} = \cos \psi \frac{\sin(\phi_s - \phi_c)}{\sin(\phi_c + \psi)},$$

where $Y = \frac{2R_0 I_{dc}}{V_{cav}}$.

Accelerating voltage $V_{acc} = V_{cav} \cos(\phi_s)$.

β is defined by the cavity design (matching with the wave guide), $\beta \simeq 3.5$.

The phase ϕ_s is defined by the energy loss/turn:
 $V_c \cos(\phi_s) = U$.

The detuning angle ψ is given by
 $\tan \psi = 2Q_l(\omega_g - \omega_c)/\omega_c$.

4.3.1 Optimum conditions

Reflected power is zero provided $\phi_c = 0$ and $\hat{I}_g = \frac{2\beta}{R_0} V_c$.
 In this case

$$\beta = 1 + Y \cos \phi_s, \quad \tan \psi = \frac{\beta - 1}{\beta + 1} \tan \phi_s.$$

Example: Some LER PEP-II parameters:

$$E = 3.1 \text{ GeV}; f_0 = 136 \text{ KHz}; f_{rf} = 496 \text{ MHz};$$

$$I_{dc} = 2.25 \text{ A}; V_{cav} = 0.85 \text{ MeV}; n_{cav} = 6.;$$

$$N_b = 5.8125 \cdot 10^{10}; s_b = 124. \text{ cm}; \phi_s = 80.9^\circ;$$

$$\beta = 3.9; Y = 18.5; \tau_d = 450.0 \text{ ns}; \psi = 74.9^\circ;$$

$$\Delta f = -145.1 \text{ KHz}; f_s = 4.5 \text{ KHz}; Q_H = 6111.1;$$

$$U_{SR} = 0.76 \text{ MeV}; P_{forward/cavity} = 40.3 \text{ kW}$$

4.4 Robinson condition of stability

Energy gain leads to the shift $z < 0$, and I_{beam} get phase

$$I_{beam}(t) = 2I_{dc} \cos(\omega t + \phi_c - \phi_s + \frac{\omega_g z}{c}).$$

Hence, ϕ_s changes, $\phi_s \rightarrow \phi_s - \frac{\omega_g z}{c}$.

I_g , I_{dc} , ψ , β do not change. Therefore, ϕ_c and Y must change. For stability, the accelerating voltage $V_{acc} \propto (1/Y) \cos \phi_s$ has to change

$$d(\cos(\phi_s)/Y)d\phi_s < 0.$$

That, after some algebra, gives Robinson criteria of stability:

$$0 < \sin(2\psi) < 2\left(\frac{1 + \beta}{Y}\right) \sin \phi_s.$$

5 F. Pedersen's analysis of stability

Small variation around the steady-state:

$$\begin{aligned}\delta V_{cav}(t) &= \frac{1}{2} \hat{V}_c e^{-i\omega_g t} [u(t) - i\Phi(t)], \\ \delta I_{tot}(t) &= \frac{1}{2} \hat{I}_{tot} e^{-i\omega_g t} [a(t) - ip(t)],\end{aligned}$$

where $u(t), \Phi(t), a(t), p(t)$ are slow functions of time.

Take Fourier transform: $a(t) \rightarrow a(\omega)$,
 $a(t)e^{-i\omega_g t} \rightarrow a(\omega - \omega_g)$.

$$\begin{aligned}\delta V_c(\omega) &= \frac{1}{2} \hat{V}_c [u(\omega - \omega_g) - i\Phi(\omega - \omega_g)], \\ \delta I_{tot}(\omega) &= \frac{1}{2} \hat{I}_{tot} [a(\omega - \omega_g) - ip(\omega - \omega_g)].\end{aligned}$$

Use $\hat{V}_c = Z_c(\omega_g)\hat{I}_{tot}$, $\delta V_c(\omega) = Z_c(\omega)\delta I_{tot}(\omega)$. Then,

$$u(\omega) - i\Phi(\omega) = \frac{Z_c(\omega + \omega_g)}{Z_c(\omega_g)} [a(\omega) - ip(\omega)].$$

In F. Pedersen notations (in Laplace transform), the transfer functions are:

$$\begin{aligned}\tilde{u}(s) &= G_{aa}\tilde{a}(s) + G_{ap}\tilde{p}(s), \\ \tilde{\Phi}(s) &= G_{pa}\tilde{a}(s) + G_{pp}\tilde{p}(s),\end{aligned}$$

where

$$\begin{aligned}G_{aa} = G_{pp} &= \text{Re}\left[\frac{\tilde{Z}_c(s - i\omega_g)}{\tilde{Z}_c(-i\omega_g)}\right], \\ G_{ap} = -G_{pa} &= \text{Im}\left[\frac{\tilde{Z}_c(s - i\omega_g)}{\tilde{Z}_c(-i\omega_g)}\right].\end{aligned}$$

To close the system, define

$$\delta I_{tot}(t) = \delta I_{gen}(t) - \delta I_{FB}(t) - \delta I_B(t).$$

The beam current at s_{cav}

$$\begin{aligned} I_{beam}(t) &= \frac{I_{dc}}{n_b c T_0} \sum_{n=1}^{n_b} \delta(ct - ns_b + \zeta_n - s_{cav}) \\ &= \frac{I_{dc}}{n_b} \sum_n \sum_k e^{\frac{2\pi i k}{c}(ct - ns_b + \zeta_n - s_{cav})} \end{aligned}$$

Retain harmonics $k = \pm n_b$ and put $\zeta_n = z(t)$, (i.e. $m = 1$ mode) and $s_{cav} = 0$:

$$I_{beam}(t) = I_{dc} e^{-i(\omega_g t + \frac{\omega_g \zeta}{c})} + c.c.$$

In the linear approximation over ζ ,

$$\delta I_b(t) = -i I_{dc} e^{-i\omega_g t} \left(\frac{\omega_g \zeta(t)}{c} \right) + c.c.$$

Consider the beam dynamics with n_{cav} cavities in the ring:

$$\frac{d^2\zeta(t)}{dt^2} + \omega_s^2\zeta(t) = -\frac{\alpha c\omega_0 n_{cav} e\delta V_c(t)}{2\pi E}.$$

Note, δV_c is taken at the arrival time

$t_b = \{(\phi_s - \phi_c)/\omega_g - \zeta(t_b)/c\} \text{mod}[T_{rev}]$ and here it can be written as $\delta V_{cav}(t) = \frac{1}{2}\hat{V}_c[u(t) - i\Phi(t)]$. That makes certain that $\zeta(t)$ is slow function of t . In frequency domain:

$$\zeta(\omega) = -\frac{\alpha c\omega_0 n_{cav}}{4\pi E} \frac{e\hat{V}_c[u(\omega) - i\Phi(\omega)]}{\omega_s^2 - \omega^2}.$$

For $\omega \simeq \omega_g$,

$$\delta I_{beam}(\omega) = i\frac{\alpha\omega_g n_{cav}\omega_0}{4\pi E} \frac{e\hat{V}_c I_{dc}[u(\omega - \omega_g) - i\Phi(\omega - \omega_g)]}{\omega_s^2 - (\omega - \omega_g)^2}.$$

The Fourier component of the FB current

$$\delta I_{FB}(\omega) = G_{FB}(\omega)\delta V_c(\omega),$$

$$R_L G_{FB}(\omega) = H_g[1 - i(\omega - \omega_c)\tau_d].$$

Using $\hat{V}_c = Z_c(\omega_g)\hat{I}_{tot}$ and the definition

$$\begin{aligned}\delta I_{tot}(\omega) &= \frac{1}{2}\hat{I}_{tot}[a(\omega - \omega_g) - ip(\omega - \omega_g)], \\ &= \delta I_{gen}(\omega) - \delta I_{FB}(\omega) - \delta I_B(\omega),\end{aligned}$$

we get

$$\begin{aligned}a(\omega) - ip(\omega) &= \frac{2\delta I_{gen}(\omega + \omega_g)}{\hat{I}_{tot}} \\ &- \frac{H_g}{R_L} [1 - i(\omega + \omega_g - \omega_c)\tau_d] Z_c(\omega_g) [u(\omega) - i\Phi(\omega)] \\ &- i \frac{\omega_s^2}{2 \sin \phi_s} \frac{Z_c(\omega_g)|\hat{I}_b|}{V_{cav}} \frac{[u(\omega) - i\Phi(\omega)]}{\omega_s^2 - \omega^2}.\end{aligned}$$

That can be written using $\hat{I}_b = 2I_{dc}e^{i(\phi_s - \phi_c)}$, and $Y = \frac{2R_0 I_{dc}}{V_{cav}}$ in the form

$$a(\omega) - ip(\omega) = \frac{2\delta I_{gen}(\omega + \omega_g)}{\hat{I}_{tot}} - T(\omega) [u(\omega) - i\Phi(\omega)],$$

$$T(\omega) = \cos(\psi)e^{i\psi} [H_g (1 - i(\omega + \omega_g - \omega_c)\tau_d)$$

$$+ \frac{i\omega_s^2 Y}{2(1 + \beta) \sin \phi_s} \frac{e^{i(\phi_s - \phi_c)}}{\omega_s^2 - \omega^2},$$

where, for n_{cav} cavities

$$\omega_s^2 = \frac{\alpha\omega_g\omega_0 e V_{cav} n_{cav} \sin(\phi_s)}{2\pi E}.$$

The system of homogeneous equations

$$a(\omega) - ip(\omega) = -T(\omega)[u(\omega) - i\Phi(\omega)],$$

$$u(\omega) - i\Phi(\omega) = \frac{Z_c(\omega + \omega_g)}{Z_c(\omega_g)} [a(\omega) - ip(\omega)].$$

define eigenvalues ω . The system is stable if all eigenvalues are in the lower half plane of ω .

The system can be written as the cubic equation for $x = \omega/\omega_s$:

$$(1 - 2ix \frac{\omega_s Q_H}{\omega_g} - i \frac{Q_H}{Q_L} \tan \psi) (x^2 - 1) = \frac{iY}{2(1 + \beta)(1 + H_g) \sin(\phi_s)}$$

As it was shown above, parameters Y and ϕ_c are related,

$$\frac{1 + \beta}{Y} = \cos \psi \frac{\sin(\phi_s - \phi_c)}{\sin(\phi_c + \psi)}.$$

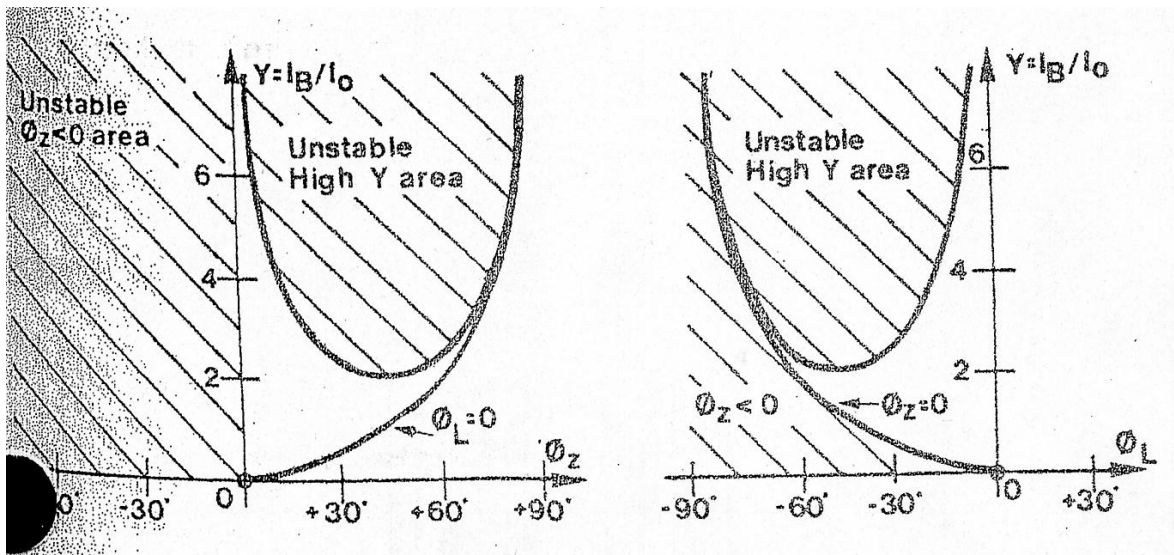


Figure 6: Stability diagram (F. Pedersen).

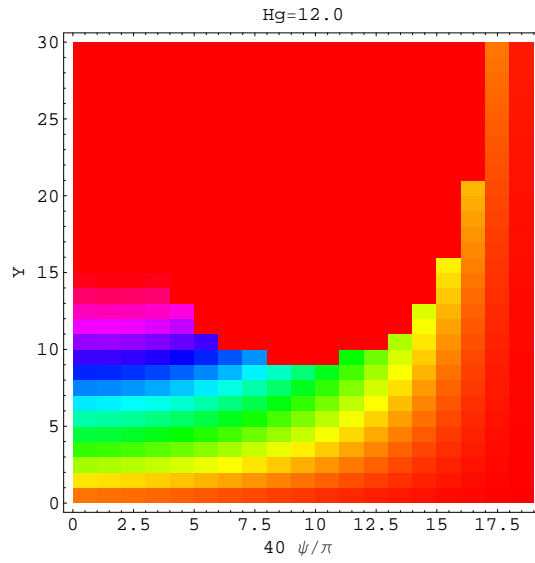


Figure 7: Stability diagram, see equation in the text.

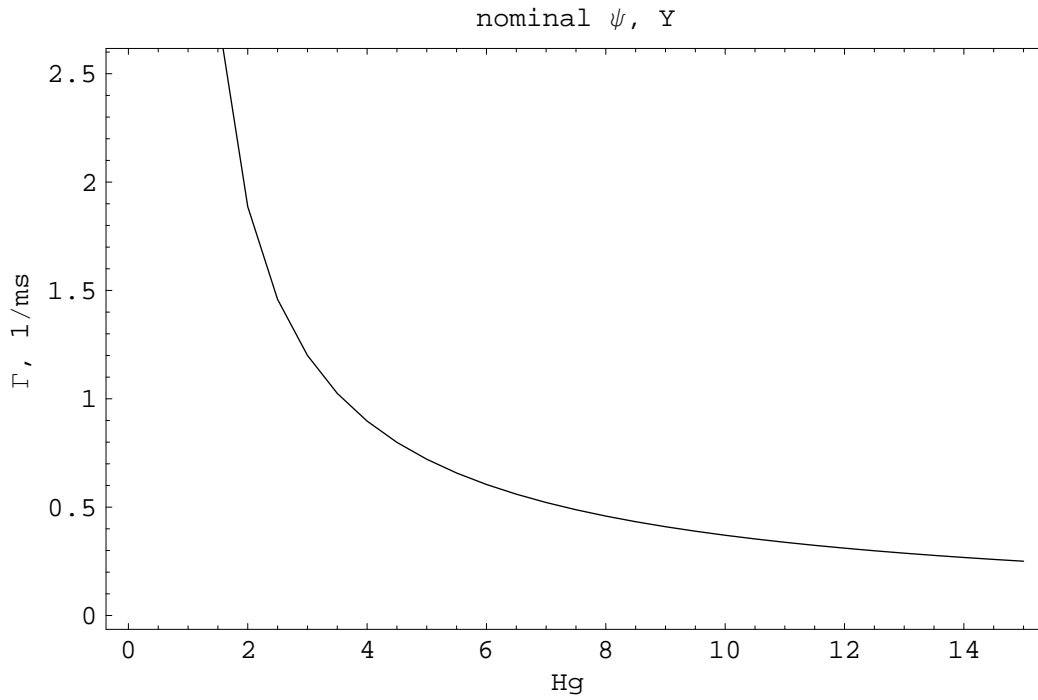


Figure 8: Growth rate vs FB gain H_g .

6 Full longitudinal FBs

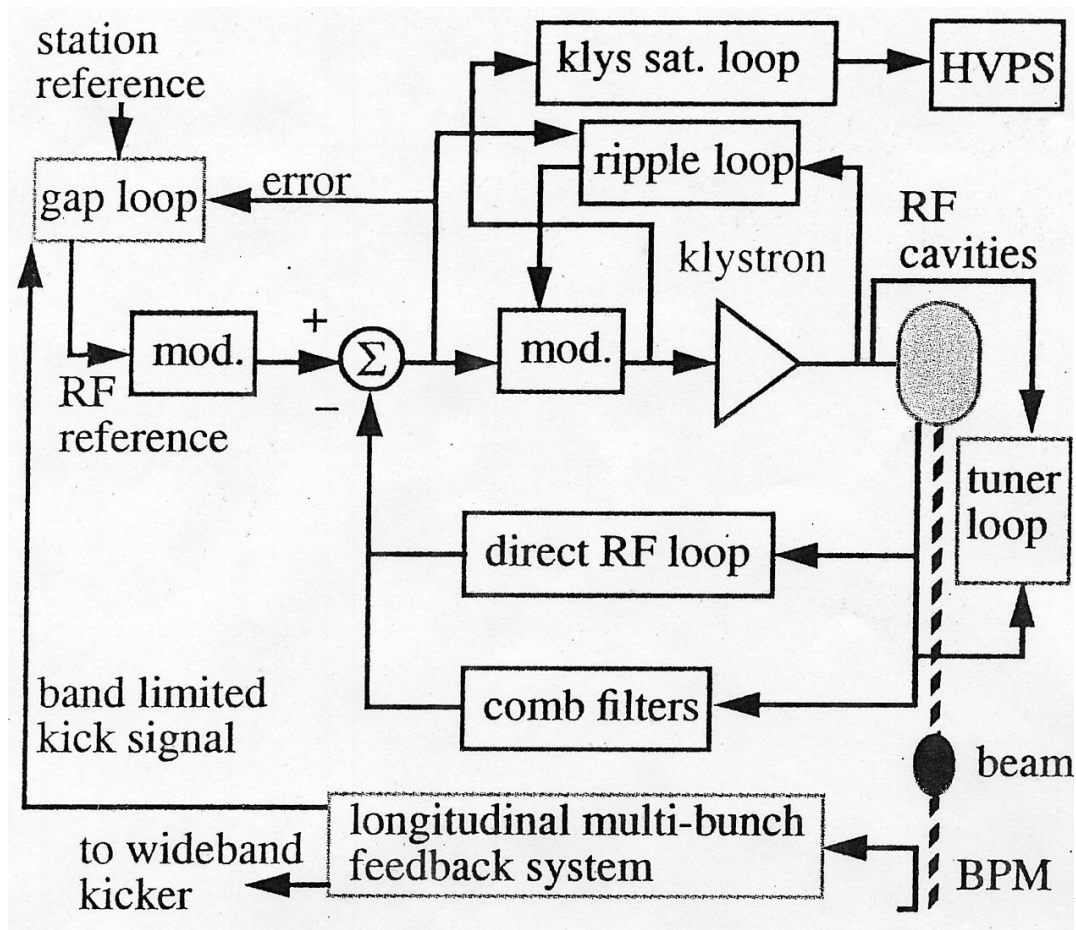


Figure 9: Full longitudinal feedback system.

6.1 Comb filter

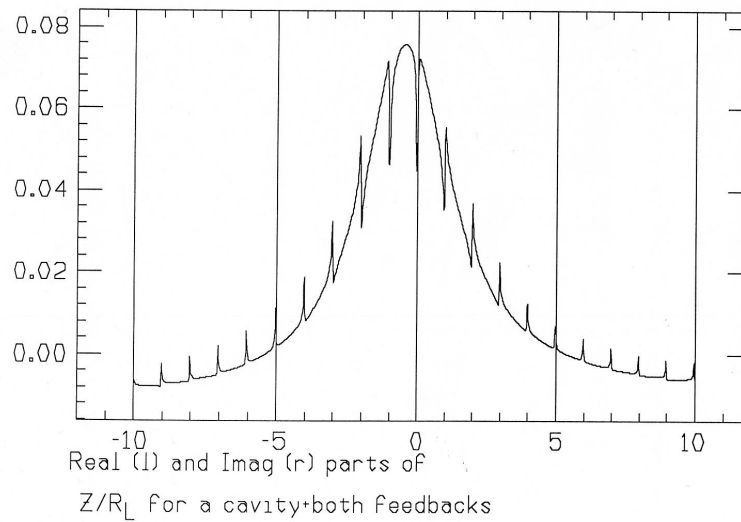


Figure 10: Real part of the transfer function of the comb filter.

6.2 Other loops

Tuner loop: minimize reflected power

Klystron saturation loop: control of the klystron operating point

ripple loop: adjust modulator to maintain constant gain/phase shift through modulator/klystron system

Gap FB loop: removes revolution harmonics from FB error signal to avoid saturation of the klystron

Woofers link: use the cavity as a kicker for low frequency modes

6.3 Noise of the FB

For longitudinal FB, see S.H. +A. Chao
SLAC-PUB-10167 (2003)

7 Bunch-by-bunch longitudinal FB

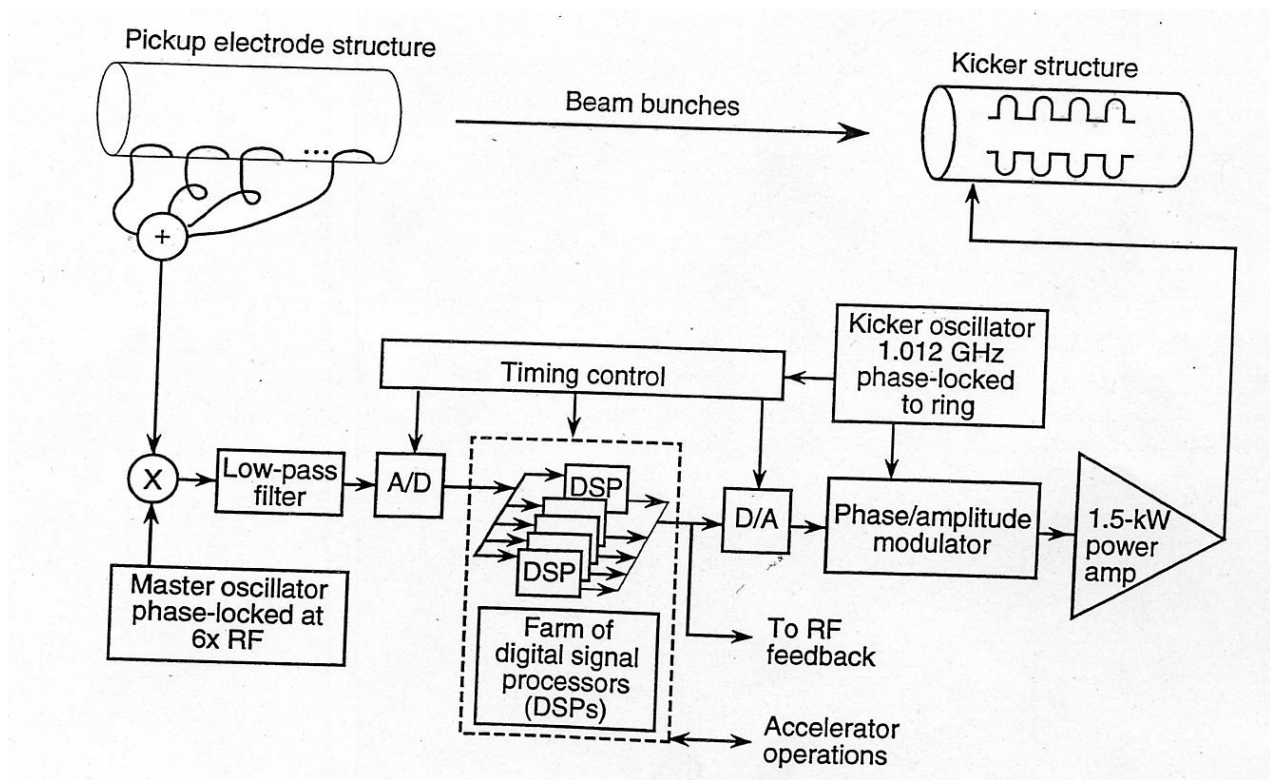


Figure 11: Schematic of the bunch-by-bunch longitudinal FB.

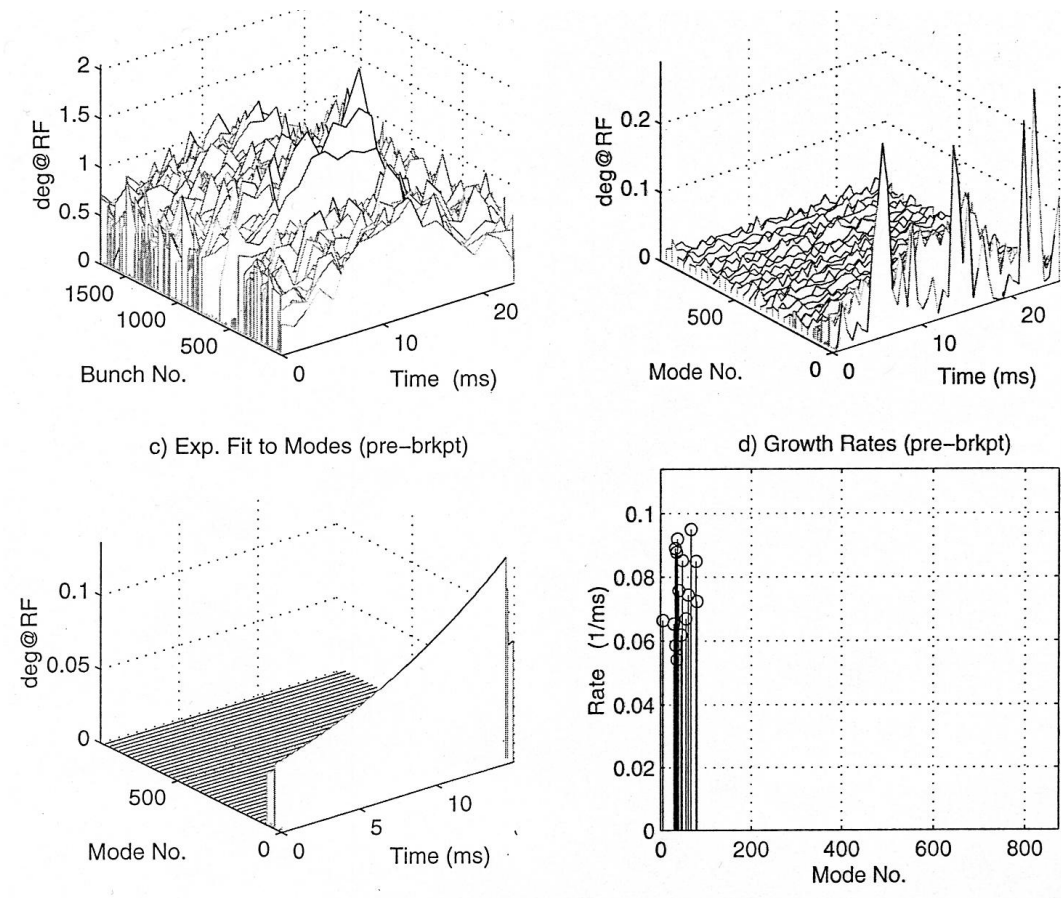


Figure 12: Diagnostics with the bunch-by-bunch longitudinal FB.

8 Transverse FB

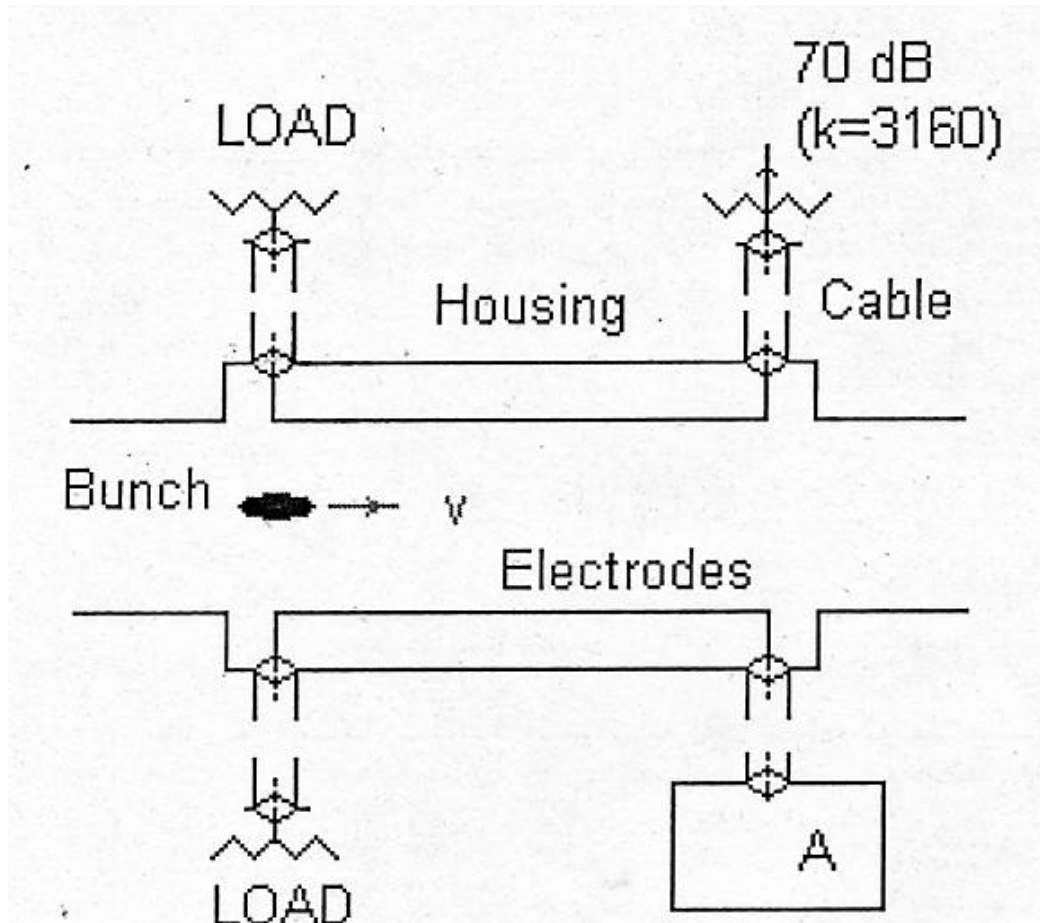


Figure 13: Schematics of the transverse FB.

However: Gproto may eliminate the DSP